DISCRIMINANT METHODS FOR BANKRUPTCY PREDICTION – THEORY AND APPLICATIONS

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Discriminant analysis consists of assigning an individual to two (or more) distinct populations, on the basis of observations of several characters of the individuals and a sample of observations of these characters from the populations. R. A. Fisher suggested a linear function of variables representing different characters, called linear discriminant function, for classifying an individual into one of the two populations. E. I. Altman adapted this approach to identify bankruptcy risk of corporations. Altman's model of bankruptcy was estimated for various countries, thereby for Polish economy. Some results of estimation and interpretation of Altman's model for Polish economy are presented in the paper. Methodological problems of discriminant analysis, especially fulfilling the basic assumptions, the analytical form of the discriminant function, the stability of the model and the estimation problems are also discussed.

***Key words:** linear discriminant function, bankruptcy models, Altman's model.*

Introduction

Private, public firm and bank failures are a serious problem for economic life. The high individual and social costs of corporate bankruptcies make the problem of bankruptcy prediction very important for managers, banks, investors, policy makers and auditors. Firms' collapses and bankruptcy appear as a serious problem of transformed economies in post-communist Central East European countries since nineties.

Bankruptcy prediction is very important, especially for three groups of recipients:

1) managers, as the most important factor for decision-making, also bookkeepers responsible for preparing financial reports on the company's activity should draw attention to the phenomena that pose a threat to the continuation of the company's operation,
2) banks, in processes of assessment of credit ability,
3) auditors, who according to the Polish Accounting Law and International Auditing Standards have to express their opinion about threats to the continuation of the company's activity.

The classical approach to bankruptcy prediction includes various rating procedures and models proposed by rating institutions. Predictions of financial distress employ also various statistical techniques, from univariate statistics to the development of multivariate statistical

Table 1. Types of bankruptcy prediction models

Derivation	*Univariate*	*Multivariate*
Iterative (simulation)		a) Experimental (credit scoring) b) Recursive partitioning c) Artificial intelligence d) Neural networking
Statistical	a) Conventional ratio analysis b) Systematic ratio analysis c) Balance sheet decomposition d) Gambler's ruin	a) Discriminant analysis b) Regression analysis c) Logit/Probit analysis d) Expanded logit e) Survival analysis
Behavioural reaction		a) Share price b) Laboratory experiments
Case studies		a) Purely descriptive b) Analysis of common factors

analysis. Bankruptcy prediction by multivariate statistical methods has become well known in the finance literature in the recent years. A milestone in the development of financial distress detection was the multivariate discriminant analysis method by Altman (1968).

McKee (2000) classified the methods and techniques of prior research on bankruptcy prediction as follows:

- Univariate ratio models
- Multiple discriminant analysis
- Linear probability models
- Multivariate conditional probability models such as Logit and Probit
- Recursive partitioning models
- Survival analysis (proportional hazard model)
- Expert systems
- Mathematical programming
- Neural networks
- Rough sets approach.

Morris (1998) presented various types of failure prediction models devised over the past 30 years (Table 1).

The aim of the paper is formulation of the problem of discriminant analysis, presentation of Fisher's linear discriminant function, its mathematical assumptions and the methods of estimation. Next, Altman's multivariate discriminant model is presented and discussed. Results of estimation of the Altman model, especially for Poland, are presented in the following part of the paper. Some methodological problems and the possibilities of other approaches are discussed at the end of the paper.

1. Linear discriminant function

In the classical approach (see, for example, Christensen (1991), Giri (1996) or Rencher (1998)), suppose we have two populations. Let $X_1 = [x_1, x_2, ..., x_{n1}]$ be the vector of n_1 observations from population 1 and $X_2 = [x_{n1+1}, x_{n1+2}, ..., x_{n1+n2}]$ the vector n_2 observations from population 2. Those are $p \times 1$ vectors, where p is the number of discriminant variables. Fisher (1936) suggested a linear function (discriminator) for classifying an individual into one of the two populations. The Fisher's discriminant method is to project these $p \times 1$ vectors to the real values via a linear function:

$$l(X) = a'X \qquad (1)$$

and try to separate the two populations as much as possible (here a is some $p \times 1$ vector).

Fisher proposed to find the vector $\hat{a}$ maximising the separation function $|S(a)|$, where:

$$S(a) = \frac{\bar{y}_1 - \bar{y}_2}{S_y} \quad (2)$$

$\bar{y}_1$ and $\bar{y}_2$ are the mean values of transformed variables Y_1 from population 1 and Y_2 from population 2,

$$S_y = \left[\frac{\sum_{i=1}^{n_1}(y_i - \bar{y}_1)^2 + \sum_{i=n1+1}^{n1+n2}(y_i - \bar{y}_2)^2}{n_1 + n_2 - 2} \right]^{\frac{1}{2}}, \text{ and } (3)$$

$$y_i = a'x_i, \; i = 1, 2, \ldots, n1 + n2. \quad (4)$$

S(a) given by (2) measures the difference between the transformed means $\bar{y}_1 - \bar{y}_2$ relative to the sample standard deviation (3). If the transformed observations $y_1, y_2, \ldots, y_{n1}$ and y_{n1+1}, y_{n1+2}, y_{n1+n2} are completely separated, $|\bar{y}_1 - \bar{y}_2|$ should be large, because the random variation of transformed data reflected by S_y is also considered.

The vector $\hat{a}$ maximising the separation $|S(a)|$ is in form:

$$\hat{a} = S_{pooled}^{-1}(\bar{x}_1 - \bar{x}_2), \quad (5)$$

where:

$$S_{pooled} = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2}, \quad (6)$$

$$S_1 = \frac{\sum_{i=1}^{n1}(x_i - \bar{x}_1)(x_i - \bar{x}_2)}{n_1 - 1}, \quad (7)$$

$$S = \frac{\sum_{n1+1}^{n1+n2}(x_i - \bar{x}_1)(x_i - \bar{x}_2)}{n_2 - 1}, \quad (8)$$

and $\bar{x}_1$, $\bar{x}_2$ are the mean values of vectors 1 and 2.

Suppose we have an observation x_0. Then, based on the discriminant function (1), we can allocate this observation to some class by the following classification rule:

– allocate x_0 to population 1 if

$$\hat{y}_0 = (\bar{x}_1 - \bar{x}_2)' S_{pooled}^{-1} x_0 \geq,$$

$$\geq \frac{1}{2}(\bar{x}_1 - \bar{x}_2)' S_{pooled}^{-1}(\bar{x}_1 + \bar{x}_2) \quad (9)$$

– allocate x_0 to population 2 if

$$\hat{y}_0 = (\bar{x}_1 - \bar{x}_2)' S_{pooled}^{-1} x_0 <$$

$$< \frac{1}{2}(\bar{x}_1 - \bar{x}_2)' S_{pooled}^{-1}(\bar{x}_1 + \bar{x}_2). \quad (10)$$

In other words, if $\hat{y}_0$ is on the right side of $\frac{\bar{y}_1 + \bar{y}_2}{2}$ (closer to $\bar{y}_1$), then allocate x_0 to population 1, and vice versa.

3. Altman's models

Altman (1968) proposed the so-called Z-Score model, which was modified later on by Altman, Haldeman; Naryanan (1977) named it ZETA® credit risk model. Altman (2000) presents revised versions of these models. He chose multiple discriminant analysis as an appropriate statistical technique for classification of the objects into one of the two groups: bankrupt (distressed) and nonbankrupt (nondistressed) firms. The Z-Score model is a linear discriminant function of some measures that are objectively weighted and summed up to arrive at an overall score that then becomes the basis for classification of firms into one of the above described groups (distressed and nondistressed).

The initial sample was composed of 66 corporations, with 33 firms in each of the two groups. After the initial groups were defined and the firms selected, balance sheet and income statement data were collected. A list of potentially helpful variables (financial ratios) was complied for evaluation. The variables were classified into five standard ratio categories, including liquidity, profitability, leverage, solvency, and activity. Concrete ratios were chosen on the basis of their popularity and potential relevancy to the study.

The final discriminant function is as follows:

$$Z = 1.2\,X1 + 1.4\,X2 + 3.3\,X3 + \\ +0.6\,X4 + 1.0\,X5, \qquad (11)$$

where:

X_1 – working capital/total assets (WC/TA),

X_2 – retained earnings/total assets (RE/TA),

X_3 – earnings before interest and taxes /total assets (EBIT/TA),

X_4 – market value equity/book value of total liabilities (MVE/TL),

X_5 – sales/total assets (S/TA).

At the next stage he tested the discriminating power of the proposed model. He found the following cut-off points of variable Z:

1.81 or less – a high probability of bankruptcy (zone I – no errors in bankruptcy classification),

3.00 or above – a low probability of bankrupty (zone II – no errors in nonbankruptcy classification),

$1.81 < Z < 2.99$ – area of uncertainty (grey area).

Altman adapted the original model several times. It has been tested for various sample periods over the last 30 years. The model has been adapted for private firms' application and for emerging market credits (Altman, 2000).

Altman, Haldeman and Narayanan (1977) constructed a second-generation model with several enhancements to the original Z-Score approach. The two samples of firms consist of 53 bankrupt firms and a matched sample of 58 nonbankrupt entities examined in the period 1969–1975; 27 potential variables of financial ratios and other measures were analysed. The variables represent profitability, coverage and other earnings relative to leverage measures, liquidity, capitalisation ratios, earnings variability and a few miscellaneous measures.

After an iterative process of reducing the number of variables, seven variables have been taken into the model [Altman (2000)]:

X_1 – return on assets,

X_2 – stability of earnings,

X_3 – debt service,

X_4 – cumulative profitability,

X_5 – liquidity,

X_6 – capitalisation,

X_7 – firm size.

Since the ZETA® model is a proprietary effort, its parameters are not published.

The linear discriminant function, proposed by Altman as a tool for financial distress and bankruptcy prediction, was applied and verified in various countries. Altman's models were firstly applied to the American business environment. Beerman (1976) estimated the parameters of the disriminant function for German enterprises. Later on they were adapted for Canadian, Japanese, Australian and West Europeans economies. A review of the applications of the Altman and other models was presented, for example, by Mossman, Bell, Swartz & Turtle (1998), Ganesalingham & Kuldeep Kumar (2001), or Ho³da (2000c).

4. Results for Poland

The original Altman's model was also applied in the 1990s in Poland. However, the applicability of this model is strongly limited. The original model includes data which are often impossible to obtain for Polish economy. Moreover, the discriminant function should be limited to the country in which the data used for developing the given model was collected. It is important largely due to the specificity of the accounting system in the given country.

Hadasik (1998) did the first attempt to estimate the of Altman model for Polish economy. She estimated nine linear discriminant models. Each of them was obtained from a different sample and using various variable selection procedures. Discriminant variables were selected using step-wise selection procedures from a large set of financial ratios characterising the condition of Polish firms.

As an example of results obtained in this work, the parameters of the first model (MOD-1) are presented here. The model has been estimated according to information from a sample of 22 failure firms and 22 "healthy" firms. The stepwise "forward" discriminant method has been applied. For the original financial ratios, linear discriminant function [D(W)] is as follows:

$$D(W) = -2.51\ W5 + 0.0014\ W9 - 0.0093\ W12 + 2.61\ W17. \quad (12)$$

For the standard form of financial ratios (SW), the estimated model is:

$$D(SW) = -1.25\ SW5 + 1.22\ SW9 - 1.55\ SW12 + 0.59\ SW17. \quad (13)$$

Here, as the discriminant variables the following financial ratios were taken:

W_5 – total debt ratio,
W_9 – charge turnover ratio,
W_{12} – cycle of stock replenishment,
W_{17} – stock return ratio.

The Wilks and F statistics have been calculated. They show that the financial ratios as discriminant variables are not significantly correlated.

The threshold value of the discriminat functions presented above is zero. If D(W) > 0 or D(SW) > 0, it means that the failure risk is low; when D(W) < 0 or D(SW) < 0, it means a high risk of failure. The general discriminant power for the above models is 93.1%.

Other models were built and estimated for different variants of sample capacities and dt procedures of discriminant variable selections. An economic interpretation of the results was presented. Synthetic results of this investigation have been published also by Appenzaller (1998).

The linear discriminant function approximated originally for Polish firms has been also proposed by Hołda (2001a). The "bankrupt" sample contained 40 economic units, and the "nonbankrupt" sample contained also 40 firms. The data represented the period 1993–1996. Initially, 28 various financial ratios were taken into account. After substantial and statistical analysis of the potential discriminant variables, five final discriminant variables have been chosen [Ho³da (2003):

QR – quick ratio,
DI – debt index,
ATR – assets turnover ratio,
RTA – return on total assets,
LTR – liabilities turnover ratio.

The linear discriminant function (Z_H), which minimises the number of classification errors, for Polish firms is as follows:

$$Z_H = 0.605 + 6.81 \cdot 10^{-1}\ QR - 1.96 \cdot 10^{-2}\ DI + 1.57 \cdot 10^{-1}\ ATR + 9.69 \cdot 10^{-3}\ RTA + 6.72 \cdot 10^{-4}\ LTR. \quad (14)$$

For the interpretation of the approximated value of Z_H, see (Hołda (2001b)):

$Z_H = -0.3$ – the probability of bankruptcy is high,
$-0,3 < Z_H < 0.1$ – the probability of bankruptcy is indefinite (grey area),
$Z_H = 0.1$ – the probability of bankruptcy is low.

The above model is widely applied in Poland for various investigations in the areas of financial distress determination, credit risk approximation and as an important part of audit procedures.

5. Conclusions

The problems and methods discussed above lead to the following conclusions:

1. The use of Fisher's linear discriminant function approach for failure risk estimation and prediction, proposed by Altman, was the first and the most important application of multivariate discriminant analysis in the corporate finance area. It gave a stimulus for constructing and approximating various bankruptcy models in many countries.
2. Fisher proposed his linear discriminant function under rather strong assumptions. The function required equal population covariance matrices, independence of the observation vectors and multivariate normality. Tests in discriminant analysis are fairly robust when the sample sizes are large or equal. If the sample sizes are small or unequal, they can be seriously affected by heterogeneity (Rencher, 1998). Those and some other requirements evoked a discussion about the appropriateness of discriminant function as a tool for bankruptcy prediction.
3. Fisher's original discriminant function was linear. However, as an extension of discriminant methods, the quadratic and polynomial discrimination functions were proposed. A problem has arisen which analytical form of the discriminant function is the best for bankruptcy prediction.
4. Altman's model was the first statistical model of corporate bankruptcy. However, its methodological and practical disadvantages inspired alternative proposals. Interesting approaches among them are (Morris, 1998) multivariate conditional probability models such as logit and probit models, expanded logit models, recursive partitioning models, survival analysis, proportional hazard model, neural networks, expert systems.

REFERENCES

1. Altman E. I. (1968); "Financial ratios, discriminant analysis and prediction of corporate bankruptcy", *Journal of Finance*, Vol. 23, September.

2. Altman E. I. (2000); "Predicting financial distress of companies: revisiting the Z-Score and ZETA® models, http://pages.stern.nyu.edu/~ealtman/Zscores.PDF

3. Altman E. I., Haldeman, P. Narayanan (1977); ZETA analysis: a new model to identify bankruptcy risk of corporations, *Journal of Banking and Finance*, June.

4. Appenzaller D. (1998); Mikro i makroekonomiczne przyczyny upadłości przedsiębiorstw w Polsce, *Ruch Prawniczy, Ekonomiczny i Socjologiczny*, z. 3–4.

5. Beerman K. (1976); *Prognosemoglichkeiten von Kapitalverlusten mit Hilfe von Jahresabschlussen*, IDW Verlag, Dusseldorf.

6. Christensen R. 1991); *Linear Models for Multivariate, Time Series, and Spatial Data*, Springer, New York.

7. Fisher R. A. (1936); The use of multiple measurements in taxonomic problems, *Annals of Eugenics* 7.

8. Ganesalingham S., Kuldeep Kumar (2001); Detection of Financial Distress via Multivariate Statistical Analysis, *Managerial Finance*, Vol. 27 no. 4.

9. Giri N. C. (1996); *Multivariate Statistical Analysis*, Dekker, New York.

10. Hadasik D. (1998); Upadłość przedsiębiorstw w Polsce i metody jej prognozowania, *Zeszyty Naukowe AER w Poznaniu*, seria II, z. 153, Wydawnictwo AE Poznań.

11. Hołda A. (2001a); Prognozowanie bankructwa jednostki w warunkach gospodarki polskiej z wykorzystaniem funkcji dyskryminacyjnej Z_H, *Rachunkowość* nr 5.

12. Hołda A. (2001b); Wstępna weryfikacja skuteczności funkcji dyskryminacyjnej Z_H, *Rachunkowość* nr 10.

13. Hołda A. (2001c); Wykorzystanie analizy dyskryminacyjnej w predykcji bankructwa – doświadczenia światowe, *Zeszyty Teoretyczne Rachunkowości*, tom 5.

14. Hołda A. (2003); Going-concern principle from the auditor's point of view, in: M. Dobija (ed.); *Gen-*

eral Accounting Theory, Cracow University of Economics.

15. McKee T. E. (2000); Developing a Bankruptcy Prediction Model via Rough Sets Theory, *International Journal of Intelligent Systems in Accounting, Finance & Management* 9.

16. Morris R. (1998); Forecasting bankruptcy: How useful are failure prediction models? *Management Accounting*, May.

17. Mossman C. E., Bell G. G., Swartz L. M, Turtle H. (1998); An empirical comparison of bankruptcy models, *Financial Review*, 33.

18. Rencher A. C. (1998); *Multivariate Statistical Inference and Applications*, Wiley, New York.