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Some considerations about attractor mapping from one-dimensional signal

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1. Introduction

Attractor mapping is one of the numerical procedures for chaos and stochasticity analysis in the numerical processes. The procedure turns to be quite specific when the analysed process is one-dimensional and dynamic parameters of the system cannot be traced (e.g. dimensional displacement, velocity, acceleration, etc.). In such case the one and only information available is the signal itself.

One of commonly used attractor mapping techniques from the time series data array is the so called "interval pass-by method" [1]. Here the numerical signal is transformed into a two-dimensional object in a plane xOy in the way that axis x holds the values of the signal itself, and axis y holds the values of the signal displaced in the time domain by fixed interval length.

Following considerations show that "interval pass-by method" may cause serious attractor modifications which may lead to incorrect attractor analysis results.

2. Initial considerations

"Interval pass-by method" transforms the numerical signal

$$s = s(i),\ i = 1,N \tag{1}$$

to a n-dimensional object in a space $\{x_1, x_2, ..., x_n\}$ in the following way:

$$x_1(i) = s(i);$$

$$x_2(i) = s(i + p_1);$$

$$\dots$$

$$x_n(i) = s(i + p_{n-1}),$$

$$i = 1,(N-p)$$

$$p = \max\,(p_1,p_2,..,p_{n-1})$$

$$0 \le p_k \le N-1,\ \ k = 1,(n-1) \tag{2}$$

here N – the signal length, p_i – the lengths of the intervals.

If the signal discretization in the time domain is δ, then interval lengths in the time domain are δp_k.

For the time being the mapping of attractor in a two-dimensional space will be considered.

3. Transformation of a harmonic signal

A harmonic function could be presented as

$$s = A \sin\,[\omega\delta(i-1) + \varphi],\ \ i = 1,N\,, \tag{3}$$

here $\omega = 2\pi/\lambda$, A – the amplitude, λ – the wave length, φ – the phase of the sine wave.

Interval pass-by method transforms signal in Eq.(3) into an ellipse in the plane $\{x,y\}$:

$$y = x \cos\,(\omega\delta p) + (A^2 - x^2)^{-1/2} \sin\,(\omega\delta p). \tag{4}$$

When

$$\delta p = \lambda(n{-}1),\ \ n = 1,\ldots \tag{5}$$

the ellipse is compressed to a line on the axis $y = x$. In analogy, when

$$\delta p = \lambda/2 + \lambda(n{-}1),\ \ n = 1,\ldots \tag{6}$$

the ellipse is compressed to the axis $y = -x$. And when

$$\delta p = \lambda/4 + (\lambda/2)(n{-}1),\ \ n = 1,\ldots \tag{7}$$

the ellipse turns to be a circle.

Attractor analysis techniques usually are based on object processing in parametric subspaces, in our case – in plane $\{x,y\}$. It's quite obvious that when Eq. (5) and Eq. (6) are satisfied, the analysis results will be not correct (it's obvious if Poincare mapping is used). On the contrary, when Eq. (7) is satisfied the most "rich" presentation of the signal is enabled.

4. Mapping quality parameter

Let's define mapping quality parameter Q which represents "richness" of the signal transformation to the plane $\{x,y\}$:

$$Q = E\,/\,\pi A^2 \tag{8}$$

here E is the area of the ellipse. Naturally, $0 \le Q \le 1$. Q turns to be 1 when harmonic signal is transformed to a circle, and gets to be 0 when it is transformed to a segment on the axis $y = x$ or $y = -x$.

The area of the ellipse can be easily calculated since the main axes of ellipse are always lying on axis $y = x$ and $y = -x$ due to the symmetricity of harmonic signal. The ellipse radiuses can be expressed like:

$$A \sin(\omega\delta p) \,/\, (1 - \cos(\omega\delta p))^{-1/2};\ A \sin(\omega\delta p) \,/\, (1 + \cos(\omega\delta p))^{-1/2} \tag{9}$$

and the area of the ellipse is:

$$E = \pi A^2 \left|\sin(\omega\delta p)\right|, \tag{10}$$

Then mapping quality parameter Q is

$$Q = \left|\sin(\omega\delta p)\right|. \tag{11}$$

5. Mapping of a non harmonic signal

Each spectrum component of the Fourier transformation of the initial signal is affected by mapping quality parameter which "damps" appropriate frequencies. If now the interval length δp is freezed, the spectrum "damping" function takes the following shape:

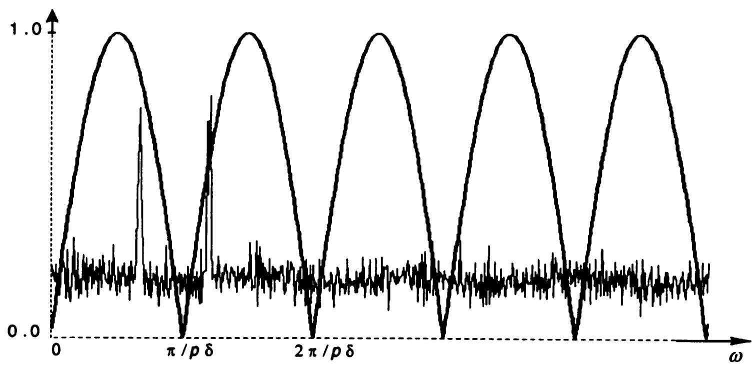

Fig. 1. Thick line stands for the spectrum damping function over the Fourier frequency spectrum density function. Here the variance of interval length enabled to secure both main frequency peaks in the areas which are not fully damped.

The "damped" Fourier spectrum looks like:

$$f_m = f\,Q(\omega) \tag{12}$$

here f – is the signal in the frequency domain transformed from the original signal s by Fourier operator Φ :

$$f = \Phi\,(s)\,. \tag{13}$$

Let's define operator M which is mapping the signal s into the plane $x0y$. Then it is obvious that

$$\mathrm{M}\,(s) = \mathrm{M}\,(\,\Phi^{-1}\,(f\,Q(\omega)))\,. \tag{14}$$

Actually, that means that quite large signal spectrum part is "lost" due to the "interval pass-by mapping method".

In fact, that is quite important when the analysed signal has some sort of chaotic modulation over more or less stationary process. In principle, variation of interval length δp can secure that at least the main peaks of Fourier spectrum are placed not very near to the nodal points of $Q(\omega)$. Some notes may be done in case of small interval length δp (this does not necessarily mean small p)

$$0 < \lim_{\omega \to 0+} (d(Q(\omega))/d\omega) = \delta p \cos(\delta p \omega) < \delta p \to 0 \,. \qquad (15)$$

This means that signal frequency domain (at least the region of reasonable interest) is totally damped, and the mapped attractor value is negligible.

On the other hand, this does not presume that large interval length δp shall be a right solution of the problem - the period of $Q(\omega)$ is $\pi / \delta p$.

So, though auto-correlation function of a random signal may guide to use very large δp, the Fourier spectrum f shall be seriously affected by $Q(\omega)$ and attractor quality shall be far from being perfect.

6. Practical notes

The imperfection of "interval pass-by method" recommends using other attractor mapping methods. Generally, if it is only possible, attractor should be built from such a number of signals what is the dimension of the attractor parametric subspace, so the information damping is avoided.

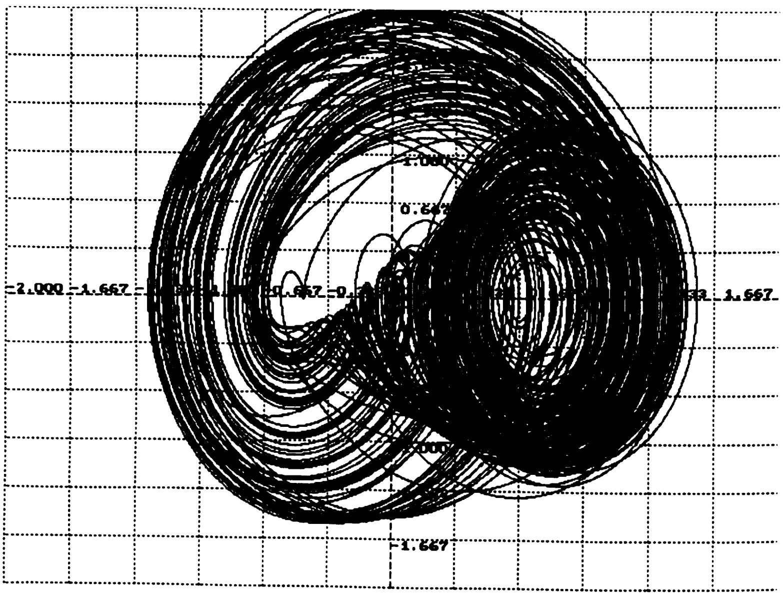

Fig. 2. The "rich" format of an attractor mapped from the numerical simulation data. X axis stands for velocity u', Y axis - acceleration u". Both signals are produced by the integration subroutine as separate data arrays.

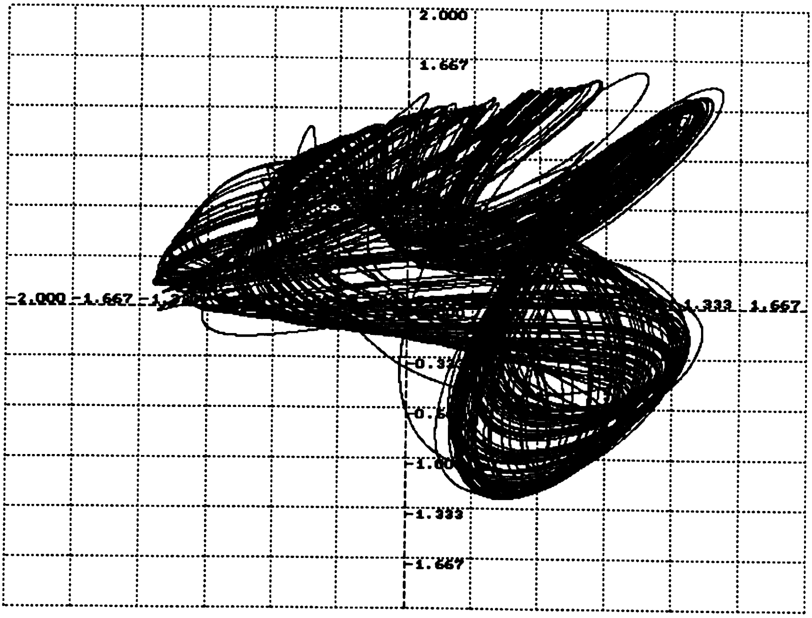

Fig. 3. The attractor of the same system as in Fig. 2. Here X axis stands for velocity u', Y axis - for $u' + p$, here p – the length of the interval, $p = \pi/4$.

This problem is not very actual in the modelling of e.g. mechanical systems - here displacement, velocity or acceleration signals are available from the modelling process even in one-dimensional body system.

The more important situation is in case of one-dimensional data arrays. Example could be a brain encephalogram or heart beat signal. Anyhow, usually such kind of signals are measured using multi sensor instruments. If the attractor mapping of such signals is considered as a mean of analysis, the use of at least two different measurement channels should give more "exact" view in comparison with an attractor built from one channel signal.

Also, it should be noted that direct numerical differentiation from one-dimensional numerical signal shall not exclude attractor information damping, i.e.

$$x(i) = s(i), \;\; y(i) = s'(i) \tag{16}$$

is in fact modified interval pass-by transformation.

Anyway, if the only available information is the signal itself, and it is necessary to build an attractor map, it would be a good idea to check how $Q(\omega)$ is damping the Fourier spectrum. The transformation interval length variation could secure more important spectrum areas, what shall give more quality for the mapped attractor.

7. Examples

Numerical modelling data of a mechanical system having chaotic mode of motion is presented in the phase plane in "rich" format (u' , u'') - Fig. 2, and "interval pass-by method" format

(u', $u' + p$) - Fig. 3. The attractors of the same system have totally different shapes, and what is most important, different characteristics. The calculated dimension of the first attractor is 1.7837, while the dimension of the second attractor is 1.3245. As the analysed signal is the same in both cases, the role of the mapping procedure turns to be of an extreme importance.

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