Liet. matem. rink., T. 40, spec. nr., 97–103

Applications of piecewise deterministic processes in reliability

Vytis KOPUSTINSKAS (VDU), Henrikas PRAGARAUSKAS (MII,VDU)
e-mail: imvyko@vdu.lt; hpraga@ktl.mii.lt

1. Introduction

Recent trends in the risk assessments of the complex industrial plants show increased interest in dynamical models arising from the coupling of the probabilistic and deterministic approaches. Conventionally used static system models, represented by the fault/event trees can not reflect dynamic behaviour of the system and complex interaction between the process variables, components and human actions.

The nature of the most complex industrial systems, like nuclear power plants (NPP) suggests that Markov type stochastic differential equations (SDEs) consisting of jump and drift components can be successfully used to represent and analyze the phenomena.

This paper discuss possible applications of the SDEs in reliability problems. In particular, Accident Localization System (ALS) of the Ignalina NPP was analyzed as a benchmark for further investigations in this area [1].

2. Dynamic reliability model

The simplified dynamics of most NPP systems can be represented by a multidimensional random process $(X_t, Y_t), t \geqslant 0$, where X_t is the hardware state process with values in a finite set $A = \{0, 1, ..., m\}$ and Y_t is the parameter state process with values in a bounded domain $B \subset \mathbb{R}^n$, whose components are physical variables like pressure, temperature, power, flux etc., that are needed to describe adequately the neutronic and thermal-hydraulic state of the reactor. It is clear that parameter state process Y_t depends on the hardware state process X_t. However, in most realistic scenarios the vice versa dependence is also important and it is not treated by the conventional PSA technique. As for example, increased temperature can influence the failure rate of the pumps.

In many cases the process $(X_t, Y_t) = (X_t^{s,x,y}, Y_t^{s,x,y})$ can be represented as a solution of the following system of SDEs

$$\begin{cases} \mathrm{d}X_t = \int c(X_t, Y_t, t, z)\, p(\mathrm{d}t\mathrm{d}z), X_s = x, \\ \mathrm{d}Y_t = b(X_t, Y_t, t)\, \mathrm{d}t, Y_s = y, \end{cases} \tag{1}$$

[1]The probabilistic safety assessment (PSA) methodology based on the standard event/fault tree analysis is widely accepted for the risk studies of the nuclear power plants and other hazardous industry [1]. Full scope Level 1 PSA analysis was completed for the Ignalina NPP-1 reactor and the plant model is further updated [2].

where $p(\mathrm{d}s\mathrm{d}z)$ is a Poisson random measure on $\mathbb{R}_+ \times \mathbb{R}$ with the compensator $\mathrm{d}t\mathrm{d}z/z^2$, the integer-valued function c defined on $A \times B \times \mathbb{R}_+ \times \mathbb{R}$ determines the jump changes of the hardware states, the function $b : A \times B \times \mathbb{R}_+ \to \mathbb{R}$ defines the rate of parameter changes, x and y are the initial states at the starting time s.

Since the dynamics of Y_t is deterministic motion between jumps of X_t, the process (X_t, Y_t) sometimes is called piecewise-deterministic [3].

One of the main problems in reliability study is the calculation of the probability that, given the accident, the parameter state process Y_t will exit from a safety domain $D \subset B$ during the accident scenario time interval $[s, T]$, i.e.,

$$v(s,x,y) = \mathbf{P}\left\{\tau^{s,x,y} \leqslant T\right\},$$

where $\tau^{s,x,y} = \inf\left\{t \geqslant s : Y_t^{s,x,y} \notin D\right\}$ and s is the accident starting time. The safety domain D is usually a polyhedron, determined by minimum or maximum values of the physical variables.

If the function v is continuously differentiable in t and y, then using Ito's formula, it is easy to verify that it satisfies the following problem

$$\begin{cases} \dfrac{\partial v(t,x,y)}{\partial t} + L\,v(t,x,y) = 0, & (t,x,y) \in (0,T) \times A \times D, \\ v(t,x,y) = 1, & (t,x,y) \in \Gamma, \\ v(T,x,y) = 0, & (x,y) \in A \times \bar{D}, \end{cases} \tag{2}$$

where $\Gamma = \{(t,x,y) \in (0,T) \times A \times \partial D : b(t,x,y) \cdot n(y) > 0\}$, $n(y)$ is the outward normal to the boundary ∂D of the safety domain D at y, and the operator L, given by

$$\begin{aligned} L\,u(t,x,y) = &\sum_{\xi \neq 0;\ x+\xi \in A} \left[u\,(t,x+\xi,y) - u\,(t,x,y)\right]\,\pi(t,x,y,\xi) + \\ &+ \sum_{i=1}^{n} b_i\,(t,x,y)\,\frac{\partial u\,(t,x,y)}{\partial y_i}, \end{aligned}$$

is the generator of the Markov process (X_t, Y_t) with the Levy measure

$$\pi(t,x,y,\xi) = \int\limits_{\{z:c(t,x,y,z)=\xi\}} \frac{\mathrm{d}z}{z^2}.$$

When v is a less regular function and does not satisfy (2) in the classical sense, in many cases it is a weak generalized solution [4] or a viscosity solution [5] to problem (2).

Other process characteristics of interest to reliability applications are the mean time of exceeding the specified limit (mean exit time), major contributors to the exit probability, uncertainty and sensitivity analysis of the input parameters.

Explicit analytical expressions for v may be obtained only in very special cases. Therefore upper and lower estimates of v, proved by combining the properties of suitably chosen sub- and super-solutions of (2), or approximate solutions of (2) are the most important in practical applications. Convergence of the approximate solutions was shown in [6], but due to the lack of space are not analyzed in this paper.

3. Application to the Ignalina NPP Accident Localization System

As the benchmark for the PDP applications in dynamic reliability one of the Ignalina Nuclear Power Plant systems – Accident Localization System (ALS) was analyzed. The ALS is the most important system, which has to ensure protection of the plant personnel and environment from the radioactive release after the ruptures in the reactor cooling system. The detailed design description of the ALS is given in [7].

Reliability of the ALS mostly depends on heat exchanger and pump system (PHEU). The ALS PHEU consists of eight heat exchangers and six pumps trains, piping and valves. Each pump train consists of one pump, three valves, two power buses and one control bus. Each heat-exchanger train consists of one heat-exchanger, two normally opened valves and service water piping.

There are two redundant valves in each side of the PHEU and all of them are operated by the same control bus 2HZ18. The system is activated by the signal, generated by various sensors. The principal diagram of the PHEU is shown in Figure 1.

Figure 2 shows results of the deterministic ALS response modeling to the design basis accident (guillotine break of the pressure header), performed by the CONTAIN code at the Lithuanian Energy Institute. It is clear that after the peak in about 4 hours is reached, the pressure will decrease only and therefore further analysis is not necessary.

Seven ALS states $A = \{0, 1, ..., 6\}$ were defined as $\min(i, j)$, where i and j are the ALS states determined by the number of operating pumps and HE respectively, as shown in Table 1.

System failures at the start moment are represented by the initial probability P_0, which was calculated using failure parameters and probabilistic plant model described in [2], i.e. failure rate λ of one PHEU pump in [2] was estimated to be 1.6×10^{-4}/h.

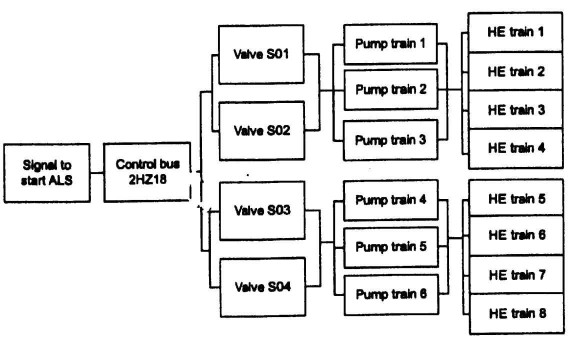

Fig. 1. Simplified diagram of the PHEU.

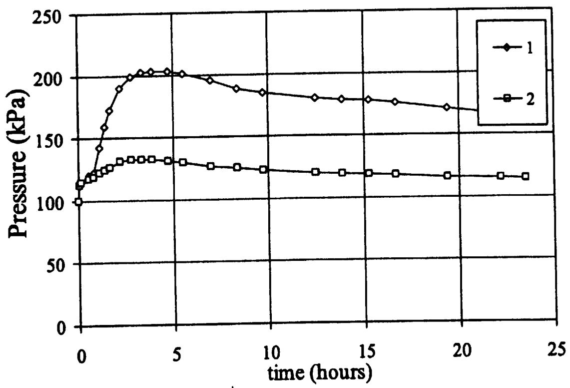

Fig. 2. Deterministic modeling of the pressure in the ALS towers. 1 – complete failure of the PHEU at the startup; 2 – no failure of the PHEU at startup.

The simplified model (1) was used for calculations with $A = \{0, 1, ..., 6\}, n = 1$, $T = 3.9$ hours, $D = (0, y_{\max})$, $y_{\max} = 103.2$ kPa, $c = c(x, z)$, $\pi(x, 1) = (6 - x)\lambda$, $\pi(x, \xi) = 0$, if $\xi \neq 1, b = b(x)$.

Note that dependence between X_t and Y_t is not included in the model due to the high complexity for analytical solution. Generator of the Markov process (X_t, Y_t) is given by

$$L\, u(x, y) = [u\,(x+1, y) - u\,(x, y)]\ (6 - x)\,\lambda + b\,(x)\,\frac{\partial u\,(x, y)}{\partial y}. \tag{3}$$

Deterministic motion was assumed to be linear and therefore $b(t, x) \equiv b(x)$ is slope coefficient as shown in Table 1 for each hardware state number x.

The main purpose of this paper was to estimate probability to exceed specified pressure limits, which could potentially lead to the release of radioactive materials outside the site and to analyze impact of random pump failures during the operation of the PHEU. The exit probability defined as $v(s, x, y) = \mathbf{P}\,\{\tau^{s,x,y} \leqslant T\}$ is the solution of previously

Table 1

System states by the minimum number of the operating trains; slope of pressure increase and initial probability P_0.

Hardware state number x	Number of pumps	Number of HE	Slope $b(x)$	P_0
0	0	0	25.8	3.60×10^{-4}
1	1	1 or 2	19.7	3.18×10^{-9}
2	2	3	15.3	5.60×10^{-7}
3	3	4 or 5	11.8	7.83×10^{-5}
4	4	6	8.3	3.37×10^{-3}
5	5	7	8.3	1.12×10^{-1}
6	6	8	8.3	8.84×10^{-1}

Table 2

Results of the exit probability calculations for the different limit values and the original system configuration.

λ factor	λ	Pressure limit $\hat{\theta}$ (above atmospheric), kPa			Event/Fault tree results
		60	70	80	
0	0	3.6×10^{-4}	3.6×10^{-4}	3.6×10^{-4}	4.4×10^{-4}
6	1×10^{-3}	3.6×10^{-4}	3.6×10^{-4}	3.6×10^{-4}	1.6×10^{-3}
60	1×10^{-2}	3.6×10^{-4}	3.6×10^{-4}	3.6×10^{-4}	1.4×10^{-1}
600	1×10^{-1}	2.6×10^{-3}	5.1×10^{-4}	4.0×10^{-4}	0.99

Table 3

Results of the exit probability calculations for the different limit values and the improved system configuration (2HZ18 control bus is excluded from the system).

λ factor	λ	Pressure limit $\hat{\theta}$ (above atmospheric), kPa			Event/Fault tree results
		60	70	80	
0	0	5.9×10^{-7}	3.4×10^{-9}	3.4×10^{-9}	8.0×10^{-5}
6	1×10^{-3}	6.0×10^{-7}	6.3×10^{-9}	3.4×10^{-9}	1.2×10^{-3}
60	1×10^{-2}	1.3×10^{-6}	1.2×10^{-7}	6.0×10^{-9}	1.4×10^{-1}
600	1×10^{-1}	2.2×10^{-3}	1.5×10^{-4}	4.5×10^{-5}	0.99

discussed equation (2). Solution of the (2) is however quite complicated even for simple examples like analysis of the PHEU. As one of the dynamic reliability advantages is the ability to encounter for failure rate dependence on the process variables (in our case, pumps failure rate dependence on the temperature). However, due to complexity of analytic solutions, this dependence was included as sensitivity case only. Absence of this dependence allowed to solve each equation of the (2) separately and recursively use the result for the next equation. Results of the calculations are shown in Tables 2 and 3.

The initial distribution of the PHEU at the startup moment clearly indicates that control bus 2HZ18 failure is the main contributor to the initial probability of the state 6 or complete failure of the PHEU at the startup. Results presented in Table 2 show that in case 2HZ18 is present in the system, exit probability is dominated by the failure of this component and impact of the pumps failures is observed only at the factor 600.

The main contributor can however be eliminated by installing the redundant or connecting to separate control buses. Therefore, the system was also analyzed without 2HZ18 control bus. Results in Table 3 show that in this case the PHEU dynamics plays more important role and at the factor of 60 and higher the rapid increase of exit probability is observed. It is shown that elimination of the single failure component in the PHEU reduces the structural ALS failure probability by at least a factor of 200. This fact can not be observed by the event/fault tree calculations, which produce the factor 5, as clear from the results shown in Tables 2 and 3.

The paper presents basic framework to analyze reliability of the technical systems dynamically. The model used to analyze the ALS of the Ignalina NPP is extremely simplified mainly due to the reason of analytic methods employed to solve (1). For more complicated models approximation schemes or Monte-Carlo should be used.

4. Conclusions

The main purpose of the paper was to demonstrate advantages of the dynamic reliability analysis against the conventional fault/event tree methods. One of the dependencies which are not encountered in the static reliability studies is component failure rate dependence on the environment parameters, usually dramatically different in accidental condition compared to the normal operation. The paper presents the basic Markovian framework to account for the inter-dependencies among different components and physical variables in the system reliability studies.

As a benchmark, the paper investigates reliability of the Ignalina nuclear power plant Accident Localization System. The results shown by this study are preliminary and can not be interpreted as final. However, the analysis of the ALS reliability clearly showed the need for further investigations to support the following conclusions

1. Control bus 2HZ18 does not satisfy single failure criteria and is the main contributor to the overall failure probability of the system. In this case startup failures are dominant and pump failures during the operation of the system impact the result only at the factor of 600, which can not be realistic.
2. The urgent improvement in order to increase reliability of the ALS is to install redundant control bus or connect different equipment to different control buses. However, if reliability of the ALS is decided to be sufficient, in-service testing procedures could be changed by risk-based methods in order to optimize operation of the plant.
3. Results indicate that elimination of the single failure component 2HZ18 in the PHEU reduces the structural ALS failure probability by at least a factor of 200, which can not be observed by the event/fault tree calculations.
4. Sensitivity study of the ALS performance indicates that in case 2HZ18 is not present the ALS dynamics plays more important role and at a factor of 60 and higher the rapid increase of exit probability is observed.

References

[1] A. Hoyland, M. Rausand, *System Reliability Theory*, John Wiley (1994).

[2] *Probabilistic Safety Assessment of the Ignalina Nuclear Power Plant. Barselina project report phase 5*, Ignalina (2000).

[3] M.H.A. Davis, *Markov Models and Optimization*, Chapman & Hall (1993).

[4] O.A. Oleinik, E.V. Radkevich, *Equations of Second Order with Non-Negative Characteristic Form*, VINITI, Moscow (1971) (in russian).

[5] M.G. Crandell, H. Ishii, P.-L. Lions, A user's guide to viscosity solutions, *Bulletin AMS*, **27**, 1–67 (1992).

[6] H. Pragarauskas, On approximation of controlled solutions of Ito equations by controlled Markov chains, *Lith. Math. J.*, **23**(1), 175–188 (1983).

[7] K. Almenas, E. Ušpuras, A. Kaliatka, *RBMK-1500 Source Book*, Kaunas (1998).

Atkarpomis deterministinių procesų taikymas patikimumo uždaviniuose

V. Kopustinskas, H. Pragarauskas

Straipsnyje nagrinėjami sudėtingų techninių sistemų patikimumo įvertinimo uždaviniai taikant stochastines diferencialines lygtis. Kaip pavyzdys, nagrinėta Ignalinos AE Avarijų Lokalizacijos Sistema.