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The applied model for the stochastic extremes

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1. Introduction

The geometrically min-stable and max-stable distributions are highly useful for solving the problems in finance, engineering, medicine [1], [3], [4]. In this paper the quality for service of the voice transference by Internet protocol is analyzed. From the experimental and theoretical investigations [2] we can make a conclusion: the transference of voice fundamentally depends on the time lag. Let the times lag T_j, $j \geqslant 1$ be the random variables whose number N is accidental for the different operators of net. We assume them to be independent variables. Suppose, the times lag have the Weibull distribution function which depends on a parameter λ. The parameter λ is a random variable distributed in accordance with the Gamma distribution law. We are interested in the distribution for the user-user minimum time lag.

Suppose $(T_1, T_2, \dots, T_N)$ is a simple random sample whose elements are random variables with the same distribution

$$F_T(t) = 1 - \mathrm{e}^{-\lambda t^{\gamma}}, \quad \lambda > 0, \ \gamma > 0, \ t \geqslant 0. \tag{1}$$

In the case of the mobile communications T is the lag time of the message transmission's operator, N is the number of operators. The parameter λ is not fixed for all operators. It is natural to assume it as random.

Let the parameter λ is a random variable distributed in accordance with the Gamma distribution law, i.e.,

$$F_\lambda(x) = \frac{\beta^{\alpha+1}}{\Gamma(\alpha+1)} \int_0^x x^{\alpha} \mathrm{e}^{-\beta x}\, \mathrm{d}x, \quad \alpha \geqslant 0, \ \beta > 0, \ \mathrm{x} > 0; \tag{2}$$

here

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} \mathrm{e}^{-t}\, \mathrm{d}t.$$

The sample size N doesn't depend onT_j, $j \geqslant 1$, and is distributed geometrically, i.e.,

$$P(N = k) = p(1-p)^{k-1}, \quad 0 < p < 1, \ k \geqslant 1. \tag{3}$$

We are interested in the distribution of the statistics

$$W_N = \min(T_1, T_2, \dots, T_N),$$

where p is fixed, or $p \to 0$. The minimal message transmission's lag time is important quality characteristic of the mobile communications ([2]).

DEFINITION. The distribution is geometrically min-stable, if there is a constant $a > 0$ such that

$$P(aW_N < t) = P(T_1 < t).$$

The distribution is asymptotically geometrically min-stable if

$$\lim_{p\to 0} P(aW_N < t) = P(T_1 < t).$$

For the telecommunication specialists, engaged in the service quality assurance at the multi-operator telecommunication environment ([2]), the most important cases are $\alpha = 0,\ \gamma > 1$. In this paper, a more general problem is analyzed. We prove that the normalized statistics W_N is either geometrically min-stable ($\alpha = 0$) or universally asymptotically geometrically min-stable ([3], [4]). Similar problems have been solved in the transference theorems ([4], [5]).

2. The main result

Denote

$$a = a(p, \alpha, \gamma) = \left(\frac{p}{1+\alpha}\right)^{-\frac{1}{\gamma}}.$$

THEOREM 1. *Suppose* (1), (2) *and* (3) *hold true. Then:*

$$1)\quad P\big(a(p,0,\gamma)\cdot W_N < t\big) = \frac{t^\gamma}{t^\gamma + \beta}, \quad t \geqslant 0.$$

$$2)\quad \lim_{p\to 0} P\big(a(p,\alpha,\gamma)\cdot W_N < t\big) = \frac{t^\gamma}{t^\gamma + \beta}, \quad t \geqslant 0.$$

Proof. By using complete probability formula, we obtain

$$\begin{aligned} P(W_N < t) &= \sum_{k=1}^{\infty} P(W_k < t) P\big(N(p) = k\big) \\ &= p\sum_{k=1}^{\infty}\Big(1 - \big(1 - P(T_1 < t)\big)^k\Big)(1-p)^{k-1} \\ &= 1 - \frac{p(1 - P(T_1 < t))}{1 - (1 - P(T_1 < t))(1-p)}. \end{aligned} \tag{4}$$

The distribution function of the component T_1 is of the form:

$$\cdot F_1(t) = \frac{\beta^{\alpha+1}}{\Gamma(\alpha+1)} \int_0^{\infty} (1 - e^{-xt^{\gamma}}) x^{\alpha} e^{-\beta x} \, dx = 1 - \left(\frac{\beta}{t^{\gamma}+\beta}\right)^{\alpha+1}, \tag{5}$$

$$\alpha \geqslant 0, \quad \beta > 0, \quad \gamma > 0.$$

Substituting $F_1(t)$ into (4), we obtain:

$$P(W_N < t) = \frac{(t^{\gamma}+\beta)^{\alpha+1} - \beta^{\alpha+1}}{(t^{\gamma}+\beta)^{\alpha+1} - \beta^{\alpha+1}(1-p)}. \tag{6}$$

Normalization of the minimum leads to the following result:

$$P\big(a(p,\alpha,\gamma)W_N < t\big) = \frac{\left(1 + \frac{t^{\gamma}p}{\beta(\alpha+1)}\right)^{\alpha+1} - 1}{\left(1 + \frac{t^{\gamma}p}{\beta(\alpha+1)}\right)^{\alpha+1} - (1-p)}. \tag{7}$$

1. For $\alpha = 0$,

$$P\big(p^{-\frac{1}{\gamma}} W_N < t\big) = \frac{1 + \frac{t^{\gamma}p}{\beta} - 1}{1 + \frac{t^{\gamma}p}{\beta} - 1 + p} = \frac{t^{\gamma}}{t^{\gamma}+\beta}.$$

The first part of the theorem is proved.

2. Obviously

$$\lim_{p\to 0} P(a(p,\alpha,\gamma)W_N < t) = \lim_{p\to 0} \frac{(1 + \frac{t^{\gamma}p}{\beta(\alpha+1)})^{\alpha+1} - 1}{(1 + \frac{t^{\gamma}p}{\beta(\alpha+1)})^{\alpha+1} - (1-p)} = \frac{t^{\gamma}}{\beta + t^{\gamma}}.$$

The proof is completed.

We are giving some remarks bellow.

Remark 1. For the structure

$$Z_N = \max(T_1, T_2, \ldots, T_{N(p)}),$$

we obtain

$$P\big(p^{\frac{1}{\gamma}} Z_N < t\big) = \frac{t^{\gamma}}{t^{\gamma}+\beta},$$

provided $\alpha = 0$. In the limit transference theorem [4], [5] we should take $p = \frac{1}{n}$, as $n \to \infty$. Consequently, our theorem generalized and makes more accurate the transference theorem.

Remark 2. In the second preposition of the above theorem, the convergence rate is of order p, as $p \to 0$. This fact is very important for the telecommunication specialists, it helps to estimate the error of approximation for the small p.

Remark 3. The distribution for term of delay T_j is the geometrically min-stable as $\alpha = 0$. And the same distribution is the asymptotically geometrically min-stable, as $\alpha \neq 0$.

Remark 4. The term of delay in the Internet can't be exponential ($\gamma = 1$). We can use only Weibull distribution with the parameter $\gamma > 1$. We just need to estimate statistically the value of parameter γ.

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REZIUMĖ

J. Borisevič, A. Aksomaitis. Vienas stochastinių ekstremumų taikomasis modelis

Straipsnyje pateikiamas stochastinių minimumų modelio taikymo telekomunikacijose galimybės. Ištirtas minimalaus vėlavimo skirstinys, kai operatorių skaičius yra atsitiktinis skaičius.