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State-space models in estimating Lithuanian Business Cycle

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1. Introduction

State-space modelling is increasingly used in ecocomics. There are two main problems in macroeconomics, that can be solved using state-space modelling: unobserved variabales and time-varying parameters estimation. The techniques that emerge from this approach are very flexible and capable of handling of wide range of time series analysis ([5], [9]). All state-space models could be divided into two groups: the linear Gausian and the non-linear non-Gausian. In this paper we apply the linear Gausian state-space model. The unobserved variables, economist mostly interested in, are business cycles. The business cycle is typically defined as deviation from trend. The economic state is normally identified as the cyclical state of gross domestic product (GDP). The position of business cycle is very important for country's monetary policy as it affects future price developments. The success of identification of business cycle could provide fiscal and monetary policymakers with better notion and prediction of future state of the economy. Filter technics should be applied to estimate business cycle. For this we can apply different approaches like Hodrick Prescott, Kalman, Band–Pass filters.

Lithuanian business cycle was already estimated in research related to nominal and real convergence of new European member states to Eurozone (ECB [4], EFN [7], MNB [8]). The authors applied variety of filtering procedures for two observed variables: the quarterly GDP and monthly industrial production. However we would be interested in multiviariate filter or in identifying business cycle as common cycle of such variables as output, unemployment, inflation. This provides us with better understanding of economy and allows us to put the story behind the model.

This paper applies small structural model to identify Lithuanian economy business cycle. The applied structural model is similar to the experience of Czech National bank [1]. Unobserved variables obtained using Kalman filter. The results compared with those of the Hodrick Prescot (HP) filter.

We will start the paper by introducing state-space models. Then we will follow with structural model. Following that we will represent our empirical part. The paper will end with results and conclusions.

2. State-space model

A standard state space formulation can be represented as follows. Let $Y_t = Z_t A_t + \varepsilon_t$ be the measument equation, where Y_t is a vector of measured variables of dimension $n \times 1$, A_t is the state vector of unobserved variables of dimension $p \times 1$, Z_t is a matrix of parameters of $n \times p$ and $\varepsilon_t \sim N(0, H_t)$. The state equation is given as $A_t = T_t A_{t-1} + \eta_t$, where T_t is a matrix of parameters and $\eta_t \sim N(0, Q_t)$. The specification of the state space system is completed by two further assumptions: the initial vector A_0 has a mean a_0 and covariance matrix P_0 and the disturbances ε_t and η_t are uncorrelated with each other in all time periods and uncorrelated with the initial state [[9], [5], [6] and many other authors].

3. The structural model

In analysis the following structural model was applied

$$y_t = \bar{y}_t + ygap_t/100, \tag{1}$$

$$u_t = \bar{u}_t - ugap_t, \tag{2}$$

$$p_t = (1-\alpha_1-\alpha_2) \cdot p_{t-1} + \alpha_3 \cdot ygap_{t-1} + \alpha_1[\Delta_4 p_t^M + 100 \cdot \Delta_4 r_t^{eq}] + \alpha_2 \pi_t + \varepsilon_t^{\pi}, \tag{3}$$

$$\bar{y}_t = \bar{y}_{t-1} + \mu_{t-1} - \omega_1 \cdot \Delta\, \bar{u}_t + \varepsilon_t^{\bar{y}_t}, \tag{4}$$

$$\mu_t = \gamma_1 \cdot \mu_{t-1} + (1-\gamma_1)\, \bar{\mu} + \varepsilon_t^{\mu}, \tag{5}$$

$$ygap_t = \theta_1 \cdot ygap_{t-1} - \theta_2 \cdot igap_t - \theta_3 \cdot rgap_t + \varepsilon_t^{ygap}, \tag{6}$$

$$\bar{u}_t = \bar{u}_{t-1} + \varepsilon_t^{\bar{u}}, \tag{7}$$

$$ugap_t = \beta_1 \cdot ugap_{t-1} + \beta_2 \cdot ygap_t + \varepsilon_t^{ugap}, \tag{8}$$

where observed variables are

- y_t – the log of GDP at constant year 2000 prices;
- u_t – the unemployment rate;
- p_t – the annual inflation, excluding energy;
- π_t – the expected inflation (calculated as four quarter (one future, coincident and to past quarters) moving average of the overall annual inflation);
- p_t^M – the annual changes of import prices;

and unobserved variables as follows

- $\bar{y}_t$ – the potential level of output;
- $ygap_t$ – the output gap;
- $\bar{u}_t$ – the non-accelerating inflation rate of unemployment (NAIRU);
- $ugap_t$ – the unemployment gap;
- r_t^{eq} – the equilibrium level of log real exchange rate;
- μ_t – the growth rate of potential output;
- $igap_t$ – the real average loans interest rates gap;
- $rgap_t$ – the real exchange rate gap.

Δ_4 means the four quarter difference operator.

Eq. 1 is identity that simply defines that output y_t is a composition of the log of potential level of output $\bar{y}_t$ and the output gap $ygap_t$. The scalling is to convert the gap units to percent. Second equation is another identity which states that the unemployment rate u_t is equal to difference between NAIRU $\bar{u}_t$ and the unemployment gap $ugap_t$. According this definition we would expect positive correlation with the output gap measure. In Eq. 3 the inflation respond to expected inflation π_t and annual changes in import prices p_t^M corrected for proxy of the equilibrium level of log real exchange rate r_t^{eq}. Eq. 4 determines the dynamics of potential output $\bar{y}$. The equation of the growth rate of potential output μ_t is described as a first order, stationary autoregressive process. Eq. 6 describes the dynamics of the output gap. The gap is formulated to evolve according to a first-order autoregressive process, but allowing for effects from the real average loans interest rates gap $igap_t$ and the real exchange rate gap $rgap_t$. For this real average loans interest rates were calculated as difference between nominal interest rates and annual inflation. The Eq. 7 specifies the NAIRU as a pure random walk driven by shocks $\varepsilon_t^{\bar{u}}$. Last equation is an Okun equation that links the movements in the unemployment gap those in the output gap.

For modelling quarterly data is used. The data sample is from 1st quarter of 1997 to 4th quarter of 2004.

4. Empirical analysis

As the structural model was already described, we have to estimate or calibrate it. There is a permanent dilema which way to choose as the "truth" model does not exist. Lithuanian data sample is very short and cover a period of structural changes in the economy and changes in policy regimes. Therefore we had chosen the calibration. To begin the calibration, we need some measures of key exogenous "gaps". For this the univariate, so called the Prior Consistent (PC) filter, was used.

The measurement equation of the PC filter for any variable x is $x_t = \bar{x}_t + xgap_t$ and the transition equations

$$\bar{x}_t = \bar{x}_{t-1} + \varepsilon_t^{\bar{x}},$$
$$xgap_t = \varepsilon_t^{xgap},$$

where $\bar{x}_t$ stands for the potential level of x, $xgap_t$ is the x gap, $\varepsilon_t^{\bar{x}}$ and ε_t^{xgap} are disturbancies.

The covariance matrix of the error terms is

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{x}}} & 0 \\ 0 & \sigma^2_{\varepsilon^{xgap}} \end{pmatrix} \quad \text{or} \quad Q = \begin{pmatrix} \frac{\sigma^2_{\varepsilon^{\bar{x}}}}{\sigma^2_{\varepsilon^{xgap}}} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\lambda} & 0 \\ 0 & 1 \end{pmatrix}.$$

The initial value of state variables were set to the value of the first observation. The initial gap was set to zero. The parameter λ has been fixed to 25 in all applications (see Box 7 in [3]). We assumed that if a "large" deviation for the trend were equal to 1, then the corresponding variance value in gap terms would be 5.

Table 1. Calibrated parameters

Parameter	a_1	a_2	a_3	d_1	d_2	d_3	b_1	b_2
Value	0.2484	0.33	0.5	0.90	0.25	0.15	0.9	0.1
Estimation_CZ	0.2676	0.106	0.43	0.92	0.19	0.05	0.88	0.08
Estimation_LT	0.18	0.39	−0.08	0.60	0.25	0.08	0.62	0.20

In our analysis the calibrated parameters were used similar to Czech National bank [[1]]. Their calibrated values presented in the first row of Table 1, as the estimated values are in the second row. The estimated parameters for the economy of Lithuania are in the third row. Almost all estimated parameters are close to calibrated values with two exceptions (a_2 and a_3). a_3 was set to 0.15 instead of 0.5 as we have not found any significant corelation between proxy of output gap (obtained using PC filter) and annual inflation (excluding energy) and a_2 was set to 0.1.

Parameter v_1 was set at 0.4, based on the approximate share of labour income in total income. In economy with large structural changes we would not expect very quick convergence to it's steady-state level and the parameter c_1 was set at 0.9.

5. State-Space representation of the system

Following the model described above we can write the measument and transition equations for the system.

$$\begin{bmatrix} y_t \\ u_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ \bar{u}_t \\ ugap_t \\ p_t \\ \bar{\mu} \end{bmatrix}$$

$$\begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ \bar{u}_t \\ ugap_t \\ p_t \\ \bar{\mu} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & (1-c_1) \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & b_2 d_1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & (1-a_1-a_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ ygap_{t-1} \\ \mu_{t-1} \\ \bar{u}_{t-1} \\ ugap_{t-1} \\ p_{t-1} \\ \bar{\mu} \end{bmatrix}$$

$$
+\begin{bmatrix} 0 \\ k_t \\ 0 \\ 0 \\ l_t \\ m_t \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{y}_t} - v_1 \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ygap} \\ \varepsilon_t^{\mu} \\ \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ugap} + b_2 \varepsilon_t^{ygap} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix},
$$

where

$$
\begin{aligned}
& k_t = -d_2 \cdot igap_t - d_3 \cdot rgap_t, \\
& l_t = b_2 \cdot k_t, \\
& m_t = \alpha_1 [\Delta_4 p_t^M + 100 \cdot \Delta_4 r_t^{eq}] + \alpha_2 \pi_t.
\end{aligned}
$$

The covariance matrix of the residuals is as follows

$$
Q = \begin{bmatrix}
\sigma^2_{\varepsilon_t^{\bar{y}_t}} + v_1^2 \sigma^2_{\varepsilon_t^{\bar{u}}} & 0 & 0 & -v_1 \sigma^2_{\varepsilon_t^{\bar{u}}} & 0 & 0 & 0 \\
0 & \sigma^2_{\varepsilon_t^{ygap}} & 0 & 0 & b_2 \sigma^2_{\varepsilon_t^{ugap}} & 0 & 0 \\
0 & 0 & \sigma^2_{\varepsilon_t^{\mu}} & 0 & 0 & 0 & 0 \\
-v_1 \sigma^2_{\varepsilon_t^{\bar{u}}} & 0 & 0 & \sigma^2_{\varepsilon_t^{\bar{u}}} & 0 & 0 & 0 \\
0 & b_2 \sigma^2_{\varepsilon_t^{ugap}} & 0 & 0 & b_2^2 \sigma^2_{\varepsilon_t^{ugap}} + \sigma^2_{\varepsilon_t^{ygap}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_t^{\pi}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

The model is estimated by maximum likelihood using the support of the Kalman filter. The calculations are done in Eviews 4.1.

6. Results

We attempted to estimate Lithuanian economy business cycle in 1997–2004. According to the conducted analysis Lithuanian economy is in it's third business cycle (see Fig. 1). Full business cycle was almost 4 years (15 quarters) long. The economy started to decline in 4th quarter of 1998 and was falling out till the second quarter of 2001. This was consequence of Russian crisis then industry and agriculture export has significantly decreased to Russia [2]. After this recession the economy started to recover and it took only 5 quarters to reach positive output gap.

We found that the economy of Lithuania has been above it's potential level at the end of 2004 (see Fig. 2). The duration of being above it's potential level differ in results of Kalman and HP filter. Following HP filter Lithuanian economy has been above it's potential already from the first quarter of 2003. But Kalman filter gives different picture – from the 4th quarter of 2002 absolute value of output gap is very close to zero and from 4th quarter of 2003 the output gap value becomes significantly positive. This significant change coincides with the biggest ever observed annual growth of

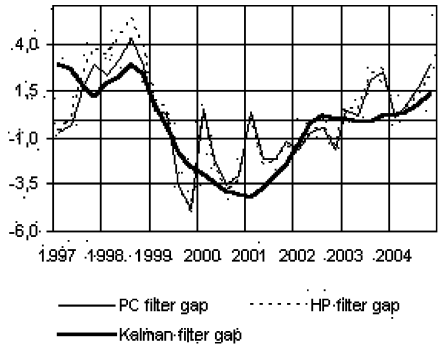

Fig. 1. Estimated business cycle.

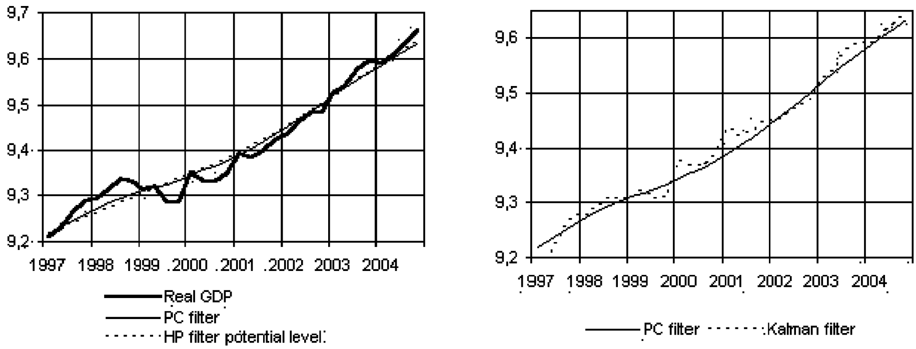

Fig. 2. Estimated potential level of the output.

Table 2. Annual output gap, % of potential level

	1997	1998	1999	2000	2001	2002	2003	2004
Kalman filter	2.1	2.4	−0.9	−3.6	−3.4	−0.3	0.0	0.7
HP filter	1.5	4.2	−1.7	−2.4	−1.5	−1.0	1.5	1.1

GDP (above 11 per cent). At the end of sample we obtained output gap value 1.34 for Kalman filter and 2.27 for HP filter. Annual output gap values are represented below (see Table 2). We see that tendency are the same for both methods, but only Kalman filter results we can consider as reasonable as we have model behind. We could interpret the last few quarters output gap growth as a result of declining unemployment and very low interest rates. Also from estimated output gap we can understand the source of racing inflation. It is driven by positive and growing output gap and expected inflation. Say, if in near future we see the growth of interest rates it will reduce output gap and this could affect inflation downward.

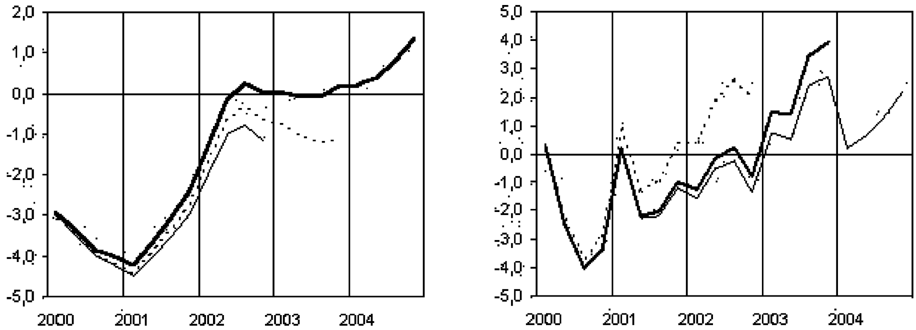

Fig. 3. The output gap stability.

We have checked the stability of our results for Kalman and HP filters. For this reason we iterated our model three times with different samples: the end of 1st sample was 4th quarter of 2002, the second – the end of 2003 and the third – 2004. The output gap results are represented in Fig. 3. We see that Kalman filter is more stable as difference at the end of 2002 is twice smaller then HP filter. The results for the end of 2003 are almost the same. We should pay attention that Kalman filter always underestimate the output gap, but HP filter tends to overestimate. This means, that structural model is less optimistic then univariate data.

7. Conclusions

Lithuanian business cycle was estimated using small structural model. Unobserved variables, output gap and potential output level, were obtained using Kalman filter. The results compared with those of the Hodrick Prescot (HP) filter. For analysis the quarterly 1997–2004 data were used. According to the results Lithuanian economy is in it's third business cycle. Full business cycle was almost 4 years (15 quarters) long. The economy was falling out from 4th quarter of 1998 till the second quarter of 2001. After this recession, that was consequence of Russian crisis, the economy started to recover and it took only 5 quarters to reach positive output gap. We received that Lithuania has been above it's potential level at the end of 2004. Following HP filter Lithuanian economy has been above it's potential already from the first quarter of 2003. But Kalman filter gave a different result. From the 4th quarter of 2002 absolute value of output gap was very close to zero and from 4th quarter of 2003 the output gap value became significantly positive. At the end of sample we obtained output gap value 1.34 for Kalman filter and 2.27 for HP filter. We see that tendencies are the same for both methods, but we only can consider Kalman filter results as reasonable as we have model behind. We have checked the stability of our results for Kalman and HP filters with different samples. Kalman filter was more stable as HP filter. Kalman filter was always underestimating the output gap, but HP filter tended to overestimate.

As there was a poor performance of inflation (Philips curve), it might be would be reasonable to try nonlinear (say square root) filter instead of linear. Also it would be interesting to make forecast from this to see how long Lithuania will stay on the strong growing path.

References

1. *Czech National Bank*, The Czech National Bank's Forecasting and Policy Analysis System (2003).
2. *Bank of Lithuania*, Annual report (1999).
3. D. Laxton, P. Isard, H. Faruqee, E. Prasad, B. Turtelboom, *MULTIMOD Mark III*: *The Core Dynamic and Steady-State Models*, IMF Occasional Paper 164 (Wasington: International Monetary Fund) (1998).
4. *European Central Bank*, The New EU Member States Convergence and Stability (2004).
5. J. Durbin, S.J. Koopman, *Time Series Analysis bys State Space Methods*, Oxford University Press (2001).
6. J. Hamilton, *Time Series Analysis*, Princeton University Press, Princeton (1994).
7. M. Artis, M. Marcellino, T. Proietti, *Characterising the Business Cycle for the Accession Countries, European Forecasting Network*, Autumn 2003 Report.
   `http://www.efn.uni-bocconi.it/Annex_to_chapter4.pdf`
8. Z. Darvas, G. Szapary, *Business Cycle Synchronization in the Enlarged EU: Comovements in the New and Old Members*, MNB Working paper 2004/1 (2004).
9. O. Basdevant, *On Applications of State-space Modelling in Macroeconomics*, Discussion Paper Series, DP2003/02, Reserve Bank of New Zealand (2003).

REZIUMĖ

A. Jakaitienė. Lietuvos verslo ciklo įvertinimas panaudojant būsenų erdvės modelius

Šio tyrimo tikslas – įvertinti Lietuvos verslo ciklą. Verslo ciklas buvo įvertintas panaudojant mažos apimties struktūrinį modelį. Struktūrinio modelio struktūra buvo parinkta atsižvelgiant į Čekijos centriniame banke naudojamą modelį verslo ciklui įvertinti. Nestebimi rodikliai, kaip gamybos apimties atotrūkis bei potenciali gamybos apimtis, buvo ivertinti panaudojant Kalman filtra. Gautieji rezultatai buvo palyginti su Hodrick Prescot (HP) filtrų gautais rezultatais. Tyrimui buvo naudojami ketvirtiniai duomenys nuo 1997 m. iki 2004 m. Atlikus tyrimą gauta, kad Lietuvos ekonomika yra trečiame verslo cikle. Nagrinėjamu laikotarpiu buvo aptiktas vienas pilnas verslo ciklas, kuris truko beveik 4 metus (viso 15 ketvirčių). Šiame cikle Lietuvos ekonomika nuo 1998 m. ketvirto ketvirčio pradėjo smukti (dėl Rusijos krizės) ir tai tęsėsi 10 ketvirčių, o atsigavimo periodas buvo gana trumpas ir tetruko vos 5 ketvirčius iki 2001 m. pirmojo ketvirčio. Pagal HP filtro rezultatus, gamybos apimties atotrūkis įgavo teigiamas reikšmes jau pirmąjį 2003 m. ketvirtį, o pagal Kalman filtro rezultatus – 2003 m. ketvirtą ketvirtį. Taip pat buvo nustatyta, kad nagrinėjamo laikotarpio pabaigai Lietuvos gamybos apimtis buvo aukščiau potencialaus lygio. 2004 metų pabaigoje gamybos apimties atotrūkis pagal Kalman filtro rezultatus buvo 1,34 procento virš potencialaus lygio, o atitinkamai pagal HP filtrą – 2,27 procento. Tyrimo rezultatų tendencijos yra panašios abiem metodais. Tačiau HP filtro rezultatai yra nulemti tiesiog duomenų filtravimo ir juos gana sudėtinga interpretuoti, tai Kalman filtro rezultatus galima daug išsamiau analizuoti, nes jie yra gauti naudojant sudarytą struktūrinį modelį. Darbe buvo taip pat patikrintas abiejųu metodų rezultatų stabilumas skirtingoms imtims. Kalman filtru gauti nestebimų komponenčių įverčiai buvo stabilesni nei įverčiai gauti HP filtru.