Long memory in high frequency foreign exchange rates: Hurst exponents dependence on data aggregation

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Abstract. This paper presents the study on long memory in absolute daily returns of the US dollar versus euro, the British pound and the Japanese yen aggregated foreign exchange rates. Pointwise, maximum price, minimum price and average price aggregation rules for high frequency foreign exchange rates are introduced. The classical R/S statistic is used to analyze Hurst exponents dependence on the choice of data aggregation function.

Keywords: long memory, Hurst exponent, high frequency foreign exchange rates, data aggregation.

1 Introduction and motivation

High frequency data is usually available for risk analysis when dealing with liquid financial instruments portfolio. In such case risk factors data sets are recorded in ticks and contain full record of transactions and their associated characteristics. One of the possible ways to deal with tick-by-tick observations is using data aggregation to obtain regular time series. Assume tick-by-tick series of financial asset prices where the ith observation consists of two variables – moment τ_i and value y_i. Time series of N such observations can be written as $\{(\tau_i, y_i)\}_{i=1}^N$. To construct regular time series the time interval between two observations δ is fixed and new time scale is obtained by taking $\tau_t^* = t\delta$, $t = 1, \ldots, N^*$. Suppose an aggregation function g is from aggregation function class $\mathcal{G}$. Regular series of financial assets prices is defined as

$$y_t^*(g) := g\big(\{(\tau_i, y_i), \tau_i \in (\tau_{t-1}^*, \tau_t^*]\}\big), \quad t = 1, \ldots, N^*.$$

There are various possible choices of aggregation functions. For example daily data can be obtained either taking close or last price of the day (equity markets) or fixing the price at a certain moment or period of the day (foreign exchange markets). Then all the rest information about price behavior during the day is not considered. The other examples of data aggregation might be taking the maximum, the minimum or the average price during the fixed period. More information about possible data aggregation is provided in the paper by Kvedaras and Račkauskas [5].

Modeling financial assets returns the basic stylized facts are considered. One of the most important stylized facts is long memory feature usually observed in absolute or squared returns. According to McNeil and Hippel [8] the covariance stationary

process y_t is said to exhibit long memory if the following condition is satisfied

$$\sum_{k=n}^{-n} |\rho_k| \to \infty, \quad n \to \infty,$$

where ρ_k is the autocorrelation function at lag k. There is a class of long memory econometric models that rather well describe financial data. Hence analyzing returns defined as a function of data aggregation g it is important to check the process long memory parameter behavior depending on the aggregation.

In this paper the long memory of absolute daily foreign exchange (FX) returns is analyzed using classical R/S statistic and Hurst exponent. FX prices were aggregated using pointwise, maximum price, minimum price and average price aggregation functions. Empirical analysis of Hurst index intraday value dependence on data aggregation function was performed for the US dollar versus euro, the British pound and the Japanese yen currencies.

2 Long memory in FX returns

According to basic stylized facts FX returns do not have long memory however absolute or square returns time series exhibit mean-averting long-term dependence (see for example [1, 2, 9]).

There are several methods for long-term dependence analysis. In this paper the Rescaled Range (R/S) analysis and Hurst exponent are used to test for long memory in high frequency FX data. The concept of rescaled range was first introduced by Hurst [4], where he investigated the river Nile level data. Mandelbrot [7] developed and popularized the classical R/S statistic methodology. Later there was a number of classical R/S statistic improvements (see for example [3, 6, 9]).

2.1 R/S statistic and Hurst exponent

The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. The R/S statistic for the returns $(X_t(g))$ depending on prices aggregation function $g \in \mathcal{G}$ can be defined as

$$[R/S](N, g) := \frac{1}{M} \sum_{t_0=1}^{M} \frac{R(N, t_0, g)}{S(N, t_0, g)},$$

where

$$R(N, t_0, g) = \max_{1 \leqslant \tau \leqslant N} \sum_{t=t_0+1}^{t_0+\tau} \left[X_t(g) - \bar{X}(N, t_0, g) \right] - \min_{1 \leqslant \tau \leqslant N} \sum_{t=t_0+1}^{t_0+\tau} \left[X_t(g) - \bar{X}(N, t_0, g) \right],$$

$$\bar{X}(N, t_0, g) = \frac{1}{N} \sum_{t=t_0+1}^{t_0+N} X_t(g)$$

and

$$S(N, t_0, g) = \left\{ \frac{1}{N} \sum_{t=t_0+1}^{t_0+N} \left[X_t(g) - \bar{X}(N, t_0, g) \right]^2 \right\}^{\frac{1}{2}}.$$

Assuming that the scaling law exists Hurst exponent H can be estimated from the following expression:

$$[R/S](N, g) \approx cN^{H(g)},$$

where c is a constant. There are three cases of Hurst exponent values that characterize different time series behavior:

- $H = 0,5$ – random walk;

- $0,5 < H < 1$ – persistent or trend reinforcing behavior;

- $0 < H < 0,5$ – anti-persistent or mean-reverting behavior.

In the case of $0,5 < H < 1$ the time series is characterized by a long memory process.

2.2 Empirical analysis

For the empirical research about Hurst exponents dependence on the chosen data aggregation function the currencies of the US dollar (USD) versus euro (EUR), the British pound (GBP) and the Japanese yen (JPY) were chosen. High frequency FX rates were available at every minute of the day over one year period. Therefore the final data set for each currency consisted of $T = 1440$ minutely rates for $N = 252$ working days of the year. To obtain daily returns δ was taken one day, then τ_t^*, $t = 1, \ldots, N$ indicates the daily time scale. Consider minutely FX rates data $\{(\tau_i, p_i)\}_{i=1}^{T \times N}$. Assume aggregation function class $\mathcal{G} = [0, 1]$. Hurst exponents were estimated using simple aggregation functions defined for each $s \in [0, 1]$:

- *pointwise aggregation*

$$p_t^{DAILY}(s) = \left\{ p_i \middle| \tau_i = \max \left\{ \tau_i \in \left(\tau_{t-1}^*, (1-s)\tau_{t-1}^* + s\tau_t^* \right] \right\} \right\}$$

- *maximum price aggregation*

$$p_t^{MAX}(s) = \max \left\{ p_i \middle| \tau_i \in \left(\tau_{t-1}^*, (1-s)\tau_{t-1}^* + s\tau_t^* \right] \right\},$$

- *minimum price aggregation*

$$p_t^{MIN}(s) = \min \left\{ p_i \middle| \tau_i \in \left(\tau_{t-1}^*, (1-s)\tau_{t-1}^* + s\tau_t^* \right] \right\},$$

- *average price aggregation*

$$p_t^{AVE}(s) = \frac{1}{m_t(s)} \sum_{\tau_i \in (\tau_{t-1}^*, (1-s)\tau_{t-1}^* + s\tau_t^*]} p_i,$$

$$m_t(s) = \#\left\{ \tau_i \in \left(\tau_{t-1}^*, (1-s)\tau_{t-1}^* + s\tau_t^* \right] \right\}.$$

Hurst exponents were estimated taking absolute returns of the aggregated prices expressed as:

$$X_t(s) = \left| \frac{\log p_t(s)}{\log p_{t-1}(s)} \right|, \quad s \in [0, 1].$$

1) EUR-USD   2) GBP-USD
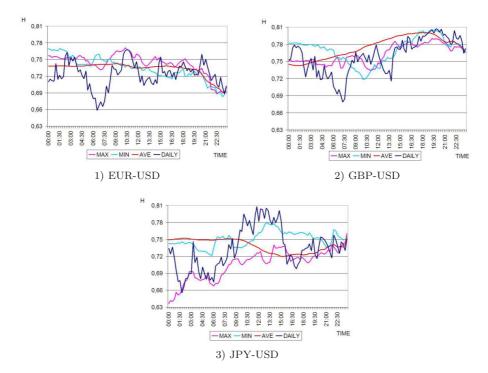
3) JPY-USD

Fig. 1. Hurst exponents dependence on data aggregation.

The results of Hurst indices estimates depending on prices aggregation function of analyzed currencies are provided in Fig. 1.

Estimates of all currencies Hurst exponents vary between 0,63 to 0,81 confirming the stylized fact about long memory in FX absolute daily returns. Pictures show that Hurst indices vary depending on the aggregation function and also change intraday with $s \in [0, 1]$. The largest fluctuations of estimated Hurst indices values are observed when pointwise aggregation function is used as the other aggregation rules give smoother prices and returns functions. The most stable intraday Hurst exponent is obtained when average aggregation function is chosen. In the end of the day all currencies Hurst indices estimated using various aggregation rules converges to the common values. Nevertheless different patterns of Hurst indices dependence on FX rates aggregation are observed for every analyzed currency. In EUR-USD currency case largest differences between various aggregation rules appear in the first part of the day. Moreover Hurst indices decrease in the second part of the day. On the contrary GBP-USD currency Hurst indices increase in the second part of the day and exhibit a slight decrease in the very end of the day. JPY-USD currency Hurst exponents have the largest variability compared to EUR-USD and GBP-USD currencies, except for average aggregation rule, where JPY-USD Hurst index is the most stable one. In conclusion, the performed analysis shows that Hurst exponents depend on the choice of data aggregation. Therefore this feature should be regarded when modeling aggregated prices returns and volatility.

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REZIUMĖ

Ilga atmintis aukšto dažnio valiutos kursuose: Hursto eksponentės priklausomybė nuo duomenų agregavimo

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Straipsnyje pateikiama euro, Didžiosios Britanijos svaro sterlingų ir Japonijos jenos JAV dolerio atžvilgiu valiutų kursų absoliučių dienos grąžų ilgos atminties analizė. Pristatomos pataškio, didžiausios kainos, mažiausios kainos ir vidutinės kainos agregavimo taisyklės aukšto dažnio valiutos kursų duomenims. Hursto eksponentės priklausomybė nuo pasirinktos agregavimo funkcijos analizuojama naudojant klasikinę R/S statistiką.

Raktiniai žodžiai: ilga atmintis, Hursto eksponentė, aukšto dažnio valiutos kursai, duomenų agregavimas.