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Adaptive joint tracking approach of system parameters and the time delay

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Abstract. The problem of adaptive joint parameters and the time delay tracking is tackled by processing of input-output observations, when nonstacionary dynamical system has an unknown time delay. The recursive approach based on the method of corrective operators is developed for their tracking. Applicability of algorithms is supported by simulation tests on a computer.

Keywords: adaptive system, parameters, tracking, estimation, time delay.

1 Introduction

Variety of techniques are worked out for the estimation of parameters and the time delay in dynamic systems, mentioned here just a few [2, 3, 4, 5, 6, 7, 8]. This paper concerns the further development of the original (c.o.) method for joint adaptive estimation and tracking of the parameters and the time delay of the system, using the prediction error model [1].

2 Statement of the problem

Consider a single input $u(k)$ and single noisy output $y(k)$ linear discrete-time time-varying system described by the difference equation

$$y(k)=\frac{B_k(q^{-1})q^{-\tau(k)}}{1+A_k(q^{-1})}u(k)+N(k)=W_k(q^{-1})u(k)+N(k)=x(k)+N(k), \tag{1}$$

where

$$A_k(q^{-1})=\sum_{i=1}^{n_a}a_i(k)q^{-i}, \qquad B_k(q^{-1})=\sum_{i=0}^{n_b}b_i(k)q^{-i} \tag{2}$$

are polynomials, which coefficient values $a_i(k)\ \overline{i=1,n_a}$, $b_i(k)\ \overline{i=0,n_b}$ and time delay $\tau(k)$ depend on k, besides, $\tau(k)\geqslant 1$ is assumed as an integer multiple of the sampling period T_s; the backward shift operator q^{-1} is defined by $q^{-1}u(k)=u(k-1)$;

$$u(k)=W_u(q^{-1})v(k)=\frac{1+F(q^{-1})}{1+G(q^{-1})}v(k), \tag{3}$$

$x(k)$ is an unmeasurable unnoisy output of the same system;

$$N(k) = W_N(q^{-1})\xi(k) = \frac{1+P(q^{-1})}{1+R(q^{-1})}\xi(k) \tag{4}$$

is an additive unmeasurable correlated noise. In (3), (4):

$$\begin{aligned} F(q^{-1}) &= \sum_{i=1}^{n_f} f_i q^{-i}, \qquad G(q^{-1}) = \sum_{i=0}^{n_g} g_i q^{-i}, \\ P(q^{-1}) &= \sum_{i=1}^{n_r} r_i q^{-i}, \qquad R(q^{-1}) = \sum_{i=0}^{n_p} p_i q^{-i} \end{aligned} \tag{5}$$

are polynomials with respective constant coefficients:

$$\boldsymbol{\theta}^{\mathbf{T}} = (\mathbf{g}^{\mathbf{T}}, \mathbf{f}^{\mathbf{T}}), \qquad \mathbf{f}^{\mathbf{T}} = (f_1, \dots, f_{\mathrm{n}_f}), \qquad \mathbf{g}^{\mathbf{T}} = (g_1, \dots, g_{\mathrm{n}_g}), \tag{6}$$

$$\boldsymbol{\alpha}^{\mathbf{T}} = (\mathbf{r}^{\mathbf{T}}, \mathbf{p}^{\mathbf{T}}), \qquad \mathbf{p}^{\mathbf{T}} = (p_1, \dots, p_{\mathrm{n}_p}), \qquad \mathbf{r}^{\mathbf{T}} = (r_1, \dots, r_{\mathrm{n}_r}). \tag{7}$$

It is assumed that all the roots of the polynomials: $A(z^{-1})$, $B(z^{-1}), P(z^{-1})$, $R(z^{-1})$, $G(z^{-1})$ and $F(z^{-1})$, where z^{-1} is the z-transform operator, are inside the unit circle on the z^{-1}-plane and the pairs of respective polynomials have no common factors. Besides, $B(z^{-1})$ has only real zeros. The true orders of polynomials are known. The input signal $u(k)$ fulfils the condition of persistent excitation of an arbitrary order according to [1]; $\xi(k)$, $v(k)$ are sequences of Gaussian variables with zero means and variances σ_ξ^2, σ_v^2.

The aim is, to track unknown time varying parameters $\boldsymbol{\Theta}^{\mathbf{T}}(k) = (\mathbf{a}^{\mathbf{T}}(k), \mathbf{b}^{\mathbf{T}}(k))$,

$$\mathbf{b}^{\mathbf{T}}(k) = (b_1(k), \dots, b_{\mathrm{n}_b}(k)), \qquad \mathbf{a}^{\mathbf{T}}(k) = (a_1(k), \dots, a_{\mathrm{n}_a}(k)), \tag{8}$$

jointly with the time delay $\tau(k)$ by processing current pairs $\{u(k), y(k)\}$.

3 Criterion to be minimized

Let us assume that parameters of polynomials $A(q^{-1})$, $B(q^{-1})$, $F(q^{-1})$ and $G(q^{-1})$ are known in advance and do not depend on k. The mean square error function for a well-known generalized prediction error model can be written as

$$Q_{in}(\varDelta\tau) = \mathcal{E}\bigl\{\bigl[1 + A\bigl(q^{-1}\bigr)\bigr]y(k) - B\bigl(q^{-1}\bigr)q^{-\hat{\tau}}u(k)\bigr\}^2 = \sigma_{\tilde{N}}^2 + 2K_{\tilde{u}\tilde{u}}(0) - 2K_{\tilde{u}\tilde{u}}(\varDelta\tau), \tag{9}$$

where $\mathcal{E}$ is the mathematical expectation; $K_{\tilde{u}\tilde{u}}(\varDelta\tau)$ denotes the autocorrelation function of the signal

$$\tilde{u}(k) = B(q^{-1})u(k), \tag{10}$$

$\sigma_{\tilde{N}}^2$ is the variance of

$$\tilde{N}(k) = \bigl[1 + A(q^{-1}\bigr]N(k), \tag{11}$$

$\varDelta\tau = \tau - \hat{\tau}$; $\hat{\tau}$ is the corresponding time delay estimate of the generalized model; τ is the true time delay; index *in* means “initial”.

The function $Q_{in}(\varDelta\tau)$ is unimodal when the roots of the characteristic equation of the transfer function $W_u(q^{-1})$ are not complex-valued. In this case the autocorrelation function $K_{\tilde{u}\tilde{u}}(\varDelta\tau)$ does not have oscilating components. The $Q_{in}(\varDelta\tau)$ will be multiextremal, on the contrary. Therefore, the minimization of the function $Q_{in}(\varDelta\tau)$ in respect of the discrete argument τ according to

$$\hat{\tau} : Q_{in}(\varDelta\tau) = \min_{\tau} Q_{in}(\varDelta\tau), \tag{12}$$

generally, will lead us to the nearest local minimum. Thus, adaptive approaches fail by the estimation of unknown parameters and the time delay. To transform a multiextremal function $Q_{in}(\varDelta\tau)$ into a unimodal one, an c.o. operator

$$W_c\big(q^{-1}\big) = B^{-1}\big(q^{-1}\big)W_u^{-1}\big(q^{-1}\big)W_e\big(q^{-1}\big), \qquad W_e(q^{-1}) = \frac{1}{\prod_{i=1}^{m}(1-\lambda_i q^{-1})} \tag{13}$$

that has no complex and negative real poles is used. Here $0 \leqslant \lambda_i < 1$, $\forall i = \overline{1,m}$, and, besides, $\lambda^{\mathbf{T}} = (\lambda_1, \ldots, \lambda_m) \neq 0$.

In practice more important is the case, when the parameters of polynomials (2), (5) are unknown. Then, it is necessary to calculate jointly the current estimates of the parameters and the time delay. Afterwards, they are substituted into the aforesaid expressions. The recursive estimates are calculated simultaneously, using the procedure, consisting of the following steps:

a) calculation parameter estimates (8) for fixed $\tau = \hat{\tau}(k)$ using the unified recursive parameter estimation algorithm;

b) calculation of estimates of parameters (6);

c) filtering of the generalized error by the c.o.:

$$e^*(k+1) = \hat{W}_{c(k+1)}(q^{-1})e(k+1), \tag{14}$$

d) calculation $\hat{\tau}(k+1)$ and estimates $(\hat{h}_1(k), \ldots, \hat{h}_{n_h}(k))^{\mathbf{T}}$ of the parameters of the noise decorelating filter:

$$\begin{bmatrix} \tau(k+1) \\ \hat{h}(k+1) \end{bmatrix} = \begin{bmatrix} \tau(k) \\ \hat{h}(k) \end{bmatrix} + \begin{bmatrix} \varDelta\tau(k) \\ \rho_k^h \varDelta h(k) \end{bmatrix}, \tag{15}$$

$$\hat{\tau}(k) = [\tau(k)], \qquad \boldsymbol{\beta}_k^{\mathbf{T}} = (\tau(k), \mathbf{h}(k)), \quad \tau_{min} \leqslant \hat{\tau}(k) \leqslant \tau_{max}, \tag{16}$$

$$\varDelta\beta_k = \begin{bmatrix} \varDelta\tau(k) \\ \varDelta\mathbf{h}(k) \end{bmatrix} = -\varLambda_k^{\beta}\boldsymbol{\nabla}_{\beta}\varepsilon(k)\varepsilon(k), \tag{17}$$

$$\varepsilon(k) = \hat{H}_k(q^{-1})e^*(k), \tag{18}$$

where

$$\varLambda_k^{\beta} = \frac{\boldsymbol{\Gamma}(k)}{\psi(k+1) + \boldsymbol{\nabla}_{\beta}^{\mathbf{T}}\varepsilon(k+1)\boldsymbol{\Gamma}(k)\boldsymbol{\nabla}_{\beta}\varepsilon(k+1)}, \tag{19}$$

$$\boldsymbol{\Gamma}(k+1) = \left\{\boldsymbol{\Gamma}(k) - \frac{\boldsymbol{\Gamma}(k)\boldsymbol{\nabla}_{\beta}\varepsilon(k+1)\boldsymbol{\nabla}_{\beta}^{\mathbf{T}}\varepsilon(k+1)\boldsymbol{\Gamma}(k)}{\psi(k+1) + \boldsymbol{\nabla}_{\beta}^{\mathbf{T}}\varepsilon(k+1)\boldsymbol{\Gamma}(k)\boldsymbol{\nabla}_{\beta}\varepsilon(k+1)}\right\}\frac{1}{\psi(k+1)}, \tag{20}$$

$$e(k+1) = y(k+1) - \sum_{i=0}^{n_b}\hat{b}_i u(k+1-i-\hat{\tau}(k)) + \sum_{i=1}^{n_a}\hat{a}_i y(k+1-i) \tag{21}$$

is the value of generalized prediction error at $k+1$ recursion, $\hat{\tau}(k)$ is the time delay estimate, calculated on the previous recursion; $\boldsymbol{\nabla}_\beta \varepsilon(k)$ is an operator of first partial derivatives; $0.95 \leqslant \psi(k) \leqslant 1$; ρ_k^h is a projecting operator of the estimates of the corresponding parameters in the admissible domain of parameters stability;

f) restoration of the parameters $\boldsymbol{\alpha}^{\mathbf{T}} = (\mathbf{r}^{\mathbf{T}}, \mathbf{p}^{\mathbf{T}})$ of the filter $W_N(q^{-1})$, generating an additive noise $N(k)$, by

$$\hat{\alpha}(k+1) = \hat{\alpha}(k) + \rho_k^\alpha \Delta\alpha(k+1), \tag{22}$$

$$\Delta\alpha(k+1) = -\Lambda_k^\alpha \bigtriangledown_\alpha \hat{\xi}(k+1)\hat{\xi}(k+1), \tag{23}$$

$$\hat{\xi}(k+1) = \frac{1+\hat{R}_k(q^{-1})}{1+\hat{P}_k(q^{-1})}\hat{N}(k+1); \tag{24}$$

g) checking elementary conditions

$$\frac{1}{t}\sqrt{\sum_{j=1}^{t} \hat{p}_j^2(k)} < \varepsilon_1, \qquad \frac{1}{n_a}\sqrt{\sum_{j=1}^{n_a} \left[\hat{r}_j(k) - \hat{a}_j(k)\right]^2} < \varepsilon_2, \tag{25}$$

assuming that $W_N(q^{-1}) = [1 + A(q^{-1})]^{-1}$, and recursive least squares (RLS) is used for tracking of unknown parameters (8). In (22) ρ_k^α by its meaning corresponds to ρ_k^h. In (25) t is the number of coefficients of the polynomial $P(q)$. Note that if (25) are satisfied, then RLS works efficiently.

4 Simulation results

The time-varying system (1) is described by the difference equation of the form

$$\left(1 + a_1(k)q^{-1} + a_2(k)q^{-2}\right)x(k) = b_0(k)q^{-\tau(k)}u(k) \tag{26}$$

with initial parameters: $b_0 = 0.4$, $a_1 = -1.26$, $a_2 = 0.52$, $\tau = 0$. Time-varying true values of coefficients of eq. (26) and that of the time delay are given in Fig. 1 in dependence of number of processed samples k. It should be noted that $b_0(k), a_1(k), a_2(k)$ values are varying until k reaches 700 observations. Delay $\tau(k)$ jumps at 100 samples. Afterwards, they all take a steady state. Their estimation and tracking are performed through both states by processing in each recursion k-th current pair of observations $\{u(k), y(k)\}$. In Fig. 1 current estimates of time-varying parameters $b_0(k)$, $a_1(k)$, $a_2(k)$, $r_1(k)$, $r_2(k)$ (part a) and $\tau(k)$ (part b), respectively, dependent on the number of processed input-output samples are given. Curves: 1–4 correspond to the true parameters $b_0(k)$, $a_1(k)$, $a_2(k)$, $\tau(k)$, respectively, while 5–8 to their estimates. Curves 9, 10 correspond to $\hat{r}_1(k)$ and $\hat{r}_2(k)$. If adaptive approach is efficient, then for large enough k distances between curves 6 and 9, 7 and 10, respectively, are small. In current case, distances are not small. Nevertheless, the algorithm tracks the time delay efficient enough.

For a tracking and estimation parameters of linear dynamical non-stationary systems with time-varying unknown pure time delay it is possible to use unified algorithms if the schema with the corrective operator is applied. In this approach both parameters and time delay in a nonstationary system are tracked and estimated simultaneously.

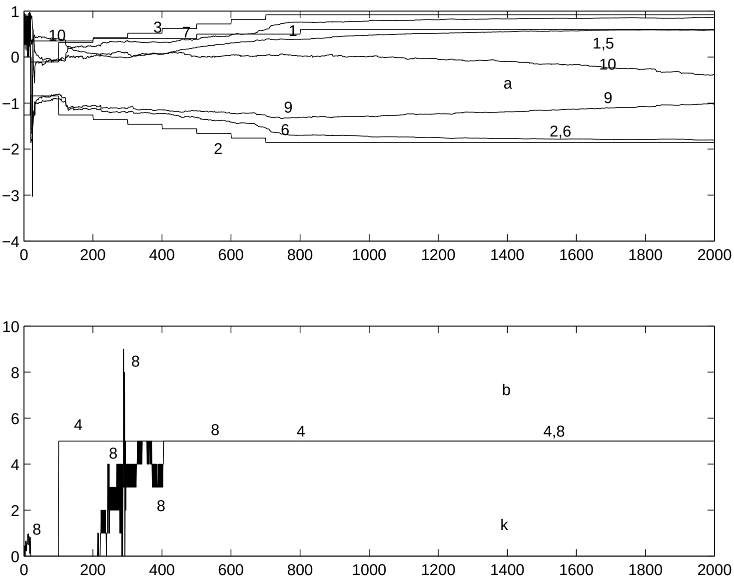

Fig. 1. Tracking of parameters and the time delay by c.o. algorithm. Input is realization of II order AR process. As $N(k)$ there was generated white noise signal $\xi(k) \sim \mathcal{N}(0, \sigma_1^2)$. Axes: Y – values of $\hat{b}_0(k), \hat{a}_1(k), \hat{a}_2(k)$, $\hat{g}_1(k), \hat{g}_2(k)$ (part a); values of $\hat{\tau}(k)$ (part b); X – numbers of processed samples (a, b)

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REZIUMĖ

Adaptyvusis jungtinis sistemos parametrų ir vėlavimo sekimo metodas
R. Pupeikis

Sprendžiamas adaptyviojo jungtinio nestacionarios sistemos nežinomų parametrų bei vėlavimo sekimo pagal stebėjimus uždavinys. Pasiūlytas rekurentinis metodas, grindžiamas koreguojančio operatoriaus taikymu. Pateikti kompiuterinio modeliavimo bei parametrų sekimo rezultatai.

Raktiniai žodžiai: adaptyvioji sistema; parametrai; sekimas; vėlavimas.