

# Notes on the historical conceptual streams for mathematics and physics teaching

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**Abstract.** Based on recent researches of mine concerning history and epistemology of sciences (physics and mathematics) one side and foundations of sciences within my physics and mathematics teaching other side, in this paper I briefly discuss and report the role played by history of science within physics and mathematics teaching. Some case-study on the relationship between mathematics, physics and logics in the history and teaching process are presented, as well.

**Keywords:** modelling, mathematics, physics, history of foundations, historical epistemology of science.

## 1 An outline

Current school science curricula have been constructed for the purpose of preparing students for university and college scientific degrees. Such education does not meet the needs of the majority of students who will not pursue tertiary studies in science or even science-related fields. These students require knowledge of the main ideas and methodologies of science. It seems that the didactics of scientific disciplines across Europe have failed to solve the crisis between scientific education and European social and economic development. This is generally recognized in the reports published concerning science education in Europe (Rocard report, etc.) which propose new strategies to be implemented in teaching through the identification and promotion of *Inquiry based Science Education* (IBSE) and other strategies. It is timely that there is a multidisciplinary dialogue exchanging new ideas and proposals between educational researchers, historians, philosophers and learning theorists.

Generally, the didactics of scientific disciplines rarely recognizes that science is an important component of European cultural heritage and provides the only well-grounded explanations we have of structures, events, and processes in the material and social world. Clearly some understanding of the practices and processes of science is essential to engage with many of the issues confronting contemporary society. However, in recent times fewer young people seem to be interested in science and technical subjects:

The loss of truth, the constantly increasing complexity of mathematics and science, and the uncertainty about which approach to mathematics is secure have caused most mathematicians [and scientists] to abandon science. With the “plague on all your horses” they have retreated to specialties in areas of

mathematics [and physics] where the methods of proof seem to be safe. They also find problems concocted by humans more appealing and manageable than those posed by nature. The crises and conflicts over what sound mathematics is have discouraged the application of mathematical methodology to many areas of our culture such as philosophy, political science, ethics, and aesthetics. The hope of finding objective, infallible laws and standards has faded. The Age of Reason is gone. With the loss of truth, man lost his intellectual center, his frame of reference, the established authority for all thought. The “pride of human reason” suffered a fall which brought down with it the house of truth. The lesson of history is that our firmest convictions are not to be asserted dogmatically; in fact they should be most suspect; they mark not our conquest but our limitations and our bounds.<sup>1</sup>

Thus(?)

Revolution in Science Education: Put Physics First!<sup>2</sup>

## 2 What about the role played by history in science teaching?

It seems that the didactics of mathematics and physics have generally proceeded according to the strict assumption that in schools (especially secondary high schools) the teaching of a significant amount of mathematical and physical principles, as well as of mere experiments, is required, regardless of their role in the learning process. This type of teaching seems to be of natural inheritance and, let us say, the consequence of a typically mechanistic-positivist perspective and, in some aspects, of mechanistic science.

A great deal of European countries and education centres are focusing especially on physics and mathematics. Many European education centres and history of science institutions like the symposia presented in European Society for the History of Science congresses, and the *Inter-Divisional Teaching Commission of the Division of Logic, Methodology and Philosophy of Science* (DLMPS) and the *International Union for History and Philosophy of Science* (IUHPS) are reflecting brilliantly upon higher scientific education and its improvements at the secondary level. It is unthinkable to learn and understand the scientific sense of a subject without deepening its intellectual and cultural background, e.g. its history and foundations: *how is it possible to continue teaching sciences being unaware of their origins, cultural reasons and eventual conflicts and values? And how is it possible to teach and comment on the contents and certainties of physics and mathematics as sciences without having first introduced sensible doubt regarding the inadequacy and fluidity of such sciences in particular contexts?*

Generally speaking, physical science makes use of experiments and apparatuses to observe and measure physical magnitudes: a unit of measurement is effectively a standardised quantity of a physical property, used as a factor to express occurring quantities of that property. Therefore, any value of a physical quantity is expressed as a comparison to a unit of that quantity. During and after an experiment, this apparatus may be illustrated and/or designed. This procedure is lacking in pure math-

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<sup>1</sup> [35, p. 7, line 12, p. 99, line 14]. (Author’s quotation).

<sup>2</sup> Lederman 54/9.

ematical studies. Therefore, one can claim that experiments and their illustrations can be strictly characterized by physical magnitudes to be measured. Mathematical-mental modelling of results of the experimental apparatus (experiments, data and errors) allows for the expansion of hypotheses and the establishment of some theses. By focusing on mathematics and physics, the previously cited aspects move towards a larger base of analysis which includes not only disciplinary matters but also interdisciplinary issues in philosophy, epistemology, logic and the foundations of physical and mathematical sciences. We need a concentrated effort for an interdisciplinary approach to teach and *learn* the relationship between physics and mathematics as a discipline of study. It has been noted that teachers regularly have great difficulty teaching historical and philosophical knowledge about science in ways that their students find meaningful and motivating.

Education needs to reevaluate scientific reasoning as an integral part of human (both humanistic and scientific) culture that could construct an autonomous scientific cultural trend in schools. *In this way, what about the importance of introducing the history of science as an integral part of the culture of teaching education to the extent of considering such a discipline – in turn – as an indissoluble pedagogical element of history and culture?* “To foresee the future of mathematics, the true method is to study its history and present state” (Poincaré quoted in: [35, p. 3]). It would be useful to pay particular attention to the elaboration of the teaching-learning process based on the reality observed by students (inductively), by a continuing critical reflection, e.g. by means of studying the historical foundations of modern physical and mathematical sciences.

*Turning from teaching based on principles to teaching (also) based on broad and cultural themes* would be crucial. It would mean teaching scientific education as well, which is a kind of education that poses *problems* and as far as physics is concerned, introducing it through historical and philosophical criticism as well. It would be helpful to practically support processes on a multidisciplinary or even on co-operational level, a kind of pedagogy able to reā€consider, from this point of view, the relationship between theory and experience, history and foundations. Let us think about (1) the lack of a relationship between physics and logic (Pisano 2005) . . . the organization of a scientific theory (axiomatic or problematic) and its pedagogical aspect based on planned and calculated processes, (2) when we use the term mechanical associated with a problem, model, law et al.; (3) the problems of foundations, for example in the teaching phase of the passage from mechanics to thermodynamics, is not yet completely solved; (4) teaching of the non-Euclidean geometries or of the planetary model as an introduction to the study of quantum mechanics, was born, as a matter of fact, only thanks to the fact that the old concept of trajectory was abandoned in favour of the probabilistic one; (5) the concept of infinite and infinitesimal in limits compared to measurements in a laboratory, etc. Through an educational offer enriched with the study of the foundations of physical and mathematical sciences, complete with the intelligent use of pedagogical computing technologies, a kind of teaching might be accomplished with the model of the prevailing method of the teaching-learning process mainly related to and coming from reality. It would be an attempt necessary for showing how typically chosen paths have not been unique in the history of science but very often an alternative possibility has existed. For example: statics in Jordanus de Nemore (ca. XIII century) and in Tartaglia [88, books VI–VIII], the physics-chemistry of Newton and Antoine-Laurent de Lavoisier (1743–1794), the mechanics

of Lazare-Nicolas-Marguerite Carnot (1753–1823) etc. More specifically: the second Newtonian principle is not strictly a physical law and it has very little in common with Galileo's physical laws rather than showing that the historical foundations of thermodynamics which are based on five [66] epistemological principles, more than the classical ones read in a textbook. From a cognitive-epistemological point of view (George and Velleman), people do not naturally and scientifically reason by means of deductive or inductive processes only. In this regard, scientific reasoning (Lakoff and Nunez) is not a part of our common knowledge reasoning), although we often intuitively compare events, tables etc. Instead, it was remarked that we reason mainly by the association of ideas and sometimes concepts are far from the scientific ones, e.g. heat and temperature, mass, weight and force-weights, the solar system and atomic orbital system in quantum mechanics, the kinetic model of gases and thermodynamics, parallel straight, material points et al. Therefore, the current scientific teaching system paradoxically changes the logical basis of reasoning.

A hypothetical proposal, which of course not the only one possible, could be the introduction within the educational plan of reading passages ad hoc centred on mathematics and physics to be analysed in the classroom, of the major works: Aristotle's mechanics (*mechanical problems*), Euclid (*Elements*), Archimedes (*On equilibrium of planes*), Tartaglia (*Quesiti*), Galilei (*Discorsi*), Torricelli (*Opera*), Lazare Carnot (*Essai*) Lavoiser (*Traité*) Sadi Carnot (*Réflexions*), Faraday (*Experimental Researches*) et al. Reading such passages, together with pre-arranged and effective work shared by several subjects leads to (1) *the student being placed before a problematic situation and being driven to realise the inadequacy of his/her basic knowledge with regard to problem solving* (2) *the beginning of the construction of scientific education in order to overcome such difficulties*. The result will be pedagogy according to which science education [16, 17, 63, 64] essentially means *setting and solving problems* and teaching means re-evaluating the relationship between theory and experience and between history and foundations. They could come together with well-structured and practical interdisciplinary work by means of the history of science. International debate should take into account pedagogical research on the foundations for history and learning-teaching science, discovering science teaching and informal learning activities as well. In this way, a student is the *protagonist*, both formally and informally (*hands-on*), of his *learning*. I feel the same about schools training experts, as these also should provide a setting that favours teaching research aimed at the critical re-construction of scientific meanings along with ideas, opinions and proper contents.

### 3 What about textbooks & curricula?

In the history of science we have significant examples of textbooks written by professional scholars and researchers during their teachings jobs. Therefore, their research (on foundations of science) and pedagogical aspects is presented, but not at all of them are presented in the same way; various factors are included. Without going too far back, one can consider the newly established theories of heat and thermodynamics and related textbooks used at the end of the 19th century: Jean Baptiste Joseph Fourier's (1768–1830) *Théorie analytique de la chaleur* [21] and the positivist Gabriel Lamé's (1795–1870) *Leçons sur la théorie analytique de la chaleur* [50] focusing on the physical-mathematical relationship in the theory of heat without considering ex-

perimental aspects of the scientific process of knowledge. The physical problem, e.g., between *chaleur* and *calorique* and the second principle are avoided. In Reech's *théorie général des effets dynamiques de la chaleur* [83] he adopted and generalized Sadi Carnot's (1796–1832) and Clapeyron's (1799–1864) reasoning in order to obtain a general formula from which each of the two theories (on caloric and on heat) can be derived under the right conditions. Italian scholar Paolo Ballada (1815–1888), also known as Paul de Saint-Robert, published (in French at the time) *Principes de thermodynamique* [84], one of the first textbooks on the subject that included scientific studies, as well as an historical part and the first biographical notes on Sadi Carnot and other scholars. The second principle is greatly emphasized. Zeuner, Verdet, Hirn, Combes, Clausius, Jacquier, Jamin should also be noted. More recently:

Textbooks, however, being pedagogic vehicles for the perpetuation of normal science, have to be rewritten in whole or in part whenever the language, problem-structure, or standards of normal science change. In short, they have to be rewritten in the aftermath of each scientific revolution, and, once rewritten, the inevitably disguise not only the role but the very existence of the revolutions that produced them. Textbooks thus begin by truncating the scientist's sense of his discipline's history and then proceed to supply a substitute for what they have eliminated. Characteristically, textbooks of science contain just a bit of history, either in an introductory chapter or, more often, in scattered references to the great heroes of an earlier age. From such references both students and professionals come to feel like participants in a long-standing historical tradition. Yet the textbook-derived tradition in which scientists come to sense their participation is one that, in fact, never existed.<sup>3</sup>

For example, a cultural role played by using a textbook based on the history of science and science teaching may be:

Mechanization also changed the image of the physical world. Is it important for the teaching of scientific studies?

What is the constitutive character of the science that we teach? Is it the same as that of the original theories?

Is modelling really the only unique bridge between experiments and theory?

Do we also teach the relationship between scientific theories?

We teach rigor and regularities interpreted by a kind of mathematics. Is it also possible to include irregularities among theories?

E.g.: Irreversibility in thermodynamics: if we remove friction, do we return to mechanical phenomena?

E.g.: Trajectory and probability in classical mechanics and quantum mechanics.

E.g.: The attempted axiomating in Special Relativity. . .

To sum up, one could think of:

Appealing to students for a scientific culture through the culture of history and philosophy, regardless of the sterile dichotomy between human and scientific disciplines.

All of us putting a professional teacher first: teachers that teach, research and publish . . .

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<sup>3</sup> [44, pp. 137–138, line 23].

Physics in the 20th century changed either the fundamentals of classical physics (and of science as well), or lifestyles (for better and for worse). A reflection based on a program, according to the spirit of research and inter-discipline, and pedagogically-oriented, is always to be regarded as a topic of interest that is never obvious.

Inviting a motivated and interested study of physics and mathematics through a wider historical and philosophical knowledge of epistemological criticism.

Trying to re-build the educational link between history and physics-mathematics. For example, history and epistemology, beginning at the end of the XIX century, seems to have no longer found a steady link with physics whose interpretation of a phenomenon is sometimes based on the involvement of an advanced and elaborated mathematics.

Dissemination and sharing of difficult theoretical and experiential works.

Making students understand that the history of scientific ideas is closely related to the history of techniques and of technologies; that is why they are different from one another.

Making others understand that scientists were once people studying in poor conditions.

Showing the real breakthrough of scientific discoveries through the study of the history of fundamentals, not yet influenced by the (modern) pedagogical requirements. For example: understanding the historical progression of the principles of classical thermodynamics and the common teaching of physics.

Letting students experiment with discoveries with enthusiastic astonishment through a guided iter reflection on the fundamental stages of progress and scientific thought.

## 4 Samples on physical and mathematical laws in the history

*On mathematics and logics.* For example, it is interesting to mention that Lazare Carnot utilized (L. Carnot, 1813, pp. 12–15) the *th'eorie de la compensation des erreurs* à la Berkeley<sup>4</sup> (1685–1753) for the infinitesimal calculus: “two errors [deux erreurs]” are made, which nevertheless are algebraically annulled in the end, and “[...] by a compensation of errors: which compensation, however, is a necessary and certain consequence of the operations of the calculus”.<sup>5</sup> (L. Carnot, 1832, p. 105, line 20). Berkeley (1685–1753) memorably disapproved of infinitesimals as “the ghosts of departed quantities” as he claimed in one of his most frequently quoted passages:

XXXV. [...] Whatever therefore is got by such Exponents and Proportions is to be ascribed to Fluxions: which must therefore be previously understood. And what are these Fluxions? The Velocities of evanescent Increments? And what are these same evanescent Increments? They are neither finite Quantities nor

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<sup>4</sup> Berkeley's lemma claims: “If with a View to demonstrate any Proposition, a certain Point is supposed, by virtue of which certain other Points are attained; and such supposed Point be itself afterwards destroyed or rejected by a contrary Supposition; in that case, all the other Points, attained thereby and consequent thereupon, must also be destroyed and rejected, so as from thence forward to be no more supposed or applied in the Demonstration” [3, Sects. XII–XIV].

<sup>5</sup> See [28, Chapters 6.1 and 9].

Quantities infinitely small, nor yet nothing. May we not call them the Ghosts of departed Quantities?<sup>6</sup>

Berkeley's lemma claims: "If with a View to demonstrate any Proposition, a certain Point is supposed, by virtue of which certain other Points are attained; and such supposed Point be itself afterwards destroyed or rejected by a contrary Supposition; in that case, all the other points, attained thereby and consequent thereupon, must also be destroyed and rejected, so as from thence forward to be no more supposed or applied in the Demonstration" ([3, Sects. XII–XIV]; see also: [2]).

Let's consider the function  $f(x) = x^2$ ; let's pass to the calculation of its derivative: the increment of the variable is  $dx$  (which at that time was not distinguished by  $\Delta x$ ) and the increment of the function is  $df$  (or  $\Delta f$ ), that is equal to:

$$df = f(x + dx) - f(x) = (x + dx)^2 - x^2 = x^2 + 2x dx + dx^2 - x^2 = 2x dx + dx^2.$$

The last term is then suppressed, since it is much smaller compared to the others. Dividing  $df$  by  $dx$  we obtain  $2x$ , which is exactly the derivative of the function. Berkeley applied his lemma, pointing out that at the beginning of the calculation  $dx \neq 0$  has been set; however, subsequently the calculation eliminates the infinitesimal of higher order at  $dx$ , considering it equal to zero. Therefore, either  $dx$  is zero, as stated at the end of the reasoning – but then the entire calculation is devoid of content –; or it is not zero, as stated in the middle of the reasoning – but this goes against the aforementioned lemma. This criticism allows Berkeley to suggest an original interpretation of the efficacy of this type of calculation. At the beginning,  $dx \neq 0$  is set and this, Berkeley claims, constitutes the first error because  $df$  as  $dx$  does not represent a specified number: nor does zero, nor a number different from zero. So there is no reason to include it in the only calculation that we know very well: elementary algebra. Continuing with the calculation, the last term is suppressed – which we know to be different from zero (otherwise we would have already eliminated it along with all of the  $dx$ ) – and this clearly represents an algebraic error. Since the first result is correct, this second error cancels out the first one.

*On Physics and Logics.* Generally speaking, a physical laws establish a mathematical-physical relationship among numerical-data and physical magnitudes. One of the laws of the German physicist George Simon Ohm (1787–1854) claims:

$$V = Ri \tag{1}$$

That is,  $V$  is equal to the resistance  $R$  multiplied by the current  $i$ . In physics, we know that the scientific validity of an affirmation depends on the physical system, the adopted theoretical-experimental model and the theory's field of applicability. That law (1) – belonging to physics – expresses a particular logical action according to which (X) the truth of a conjunction (compared to given model) would imply the truth of both members in (1); while the opposite would not be logically applicable. Moreover, general speaking, the (Y) non-truth of a given proposition does not imply the truth of its negation. It means that, given two quantities  $A$  and  $B$ , one can write:

$$(X) (A \wedge B) \text{ true} \Rightarrow A \text{ true}, B \text{ true}; A \text{ true}, B \text{ true} \neq (A \wedge B) \text{ true.} \tag{2}$$

$$(Y) \neg \neq \neg\neg(A). \tag{3}$$

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<sup>6</sup> [2, Sect. XXXV, p. 18, line 9].

In other words, this result would weakly violate the validity of the principle of *Tertium non datur*.<sup>7</sup> Thus, what does it mean measure by (1)? As a matter of fact, by considering up above discussion, in order to measure (1) a simultaneous measurement of the three quantities  $V$ ,  $R$ ,  $i$ , should be required. In this sense, in order for (1) to be experimentally *true*, it is necessary for three real corresponding numbers  $a$ ,  $b$ ,  $c$ , should exist respectively for the measurements of  $V$ ,  $R$ , and  $i$ . Thus, one can write:

$$a = bc. \quad (4)$$

The measurement is obviously never *perfect* (or to be more precise, the experimental data should coincide with the theoretical ones only in the limits of the experimental errors). Therefore, because of the *uncertainties* of the devices and error of measurement, in the same range of measurement of  $a$ ,  $b$ ,  $c$ , it should exist another real triad  $a'$ ,  $b'$ ,  $c'$  with  $(a' \neq a)$ ,  $(b' \neq b)$ ,  $(c' \neq c)$ , should exist, so that:

$$a' \neq b'c'. \quad (5)$$

To summaries, both (2) and its negation (3) would be true with respect to a given experimental situation. Nevertheless, the conjunction of (2) and (3) would not be true, because it should exists a real triad so that one simultaneously should obtain:

$$a = bc \wedge a \neq bc.$$

which in classical logics is false. These two examples could be express by sentence which their scientific content should follow, generally speaking, they belong to non-classical logic. An idem situation concerning with Newtonian principle of inertia [61, I, p. 2, p. 23] where the physical laws can be expressed by means of a proposition preceded by two universal and existential quantifications<sup>8</sup>:  $(\forall)$  (“for all”) and  $(\exists)$  (“there exists” or “for some”) [60, 66]. In the end, within relationship physics mathematics every physical variable should be subjected to its measurement. If the measurement cannot apply, the scientific content generates uncertainties in scientific knowledge. These historical and epistemological aspects plays have an important role in science education during, e.g. the shift from laboratory and theoretical lesions [67, 68].

## 5 Final remarks

In my opinion focusing on mathematics, physics and their relationship, a larger base of analysis should be adopted. It should include not only disciplinary matters but also interdisciplinary issues including history, historical epistemology, logic and the foundations of physical and mathematical sciences. A multidisciplinary teaching approach

<sup>7</sup> In the classical mathematical logic, three main laws are – a priori – claimed: *The principle of non-contradiction*: “ $x$  and non  $x$ ”. One thing cannot both be and not be at the same time and in the same respect. The same proposition cannot be both true and false. *The principle of Tertium non datur*: Either “ $x$ ” is “ $y$ ” or “ $x$ ” is not “ $y$ ”. Either a thing is or it is not, there is no third possibility. *The principle of identity*: Every being is that which it is. Each being is separated in its existence from other beings. On *The principle of Tertium non datur* and related epistemological topics see: [36, 37, 38]).

<sup>8</sup> At beginning of past century, Thoralf Albert Skolem (1887–1963) suggested a technique to formalize the existential quantification on  $y$ -variable of a given predicate into a constructive mathematical function [86, 32].



based on large themes-problems toward a scientific education based on different formulations of the same theory would be appreciated. Therefore some of the following case-studies on the relationship between physics and mathematics are presented and discussed. For example, the lack of a relationship between physics and logic (classical and non-classical), space and time in mechanics, mechanics and thermodynamics, *ad absurdum* proofs, non-Euclidean geometries and space in physics, the planetary model and quantum mechanics, infinite-infinitesimal and measurements in the laboratory, heat-temperature-friction, reversibility, continuum-discrete models in mechanics, ad hoc hypotheses, local-global interpretations and differential equations-integral, point-range, constructive mathematics, the kinetic model of gases and thermodynamics; also, a provocative hypothesis: *...generally speaking, we should not lose the certainty of critical thought on science... if we do not do anything, then nothing changes... but if we do something (a few crucial things), maybe something could be improved.*

In the end, this brief reflection should convey that it is urgent to establish the basis for a debate that appears ethically correct and professionally necessary. Maybe, operating in a different way, we could also contribute to building a school (or university) linked to the new perspectives of science – the image and teaching of science without limitations on specializations, pushing past disciplinary competency. Therefore:

Revolution in Science Education: Put Physics and Mathematics Curricula First!

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REZIUMĖ

**Pastabos apie matematikos ir fizikos mokymo istorines koncepcines sroves**

*R. Pisano*

Remdamasis paskutiniaisiais savo mokslo (fizikos ir matematikos) istorijos ir epistemologijos tyrimais bei moksliniu patyrimu, kurį įgijau dėstydamas fiziką ir matematiką, šiame straipsnyje aš trumpai nagrinėju mokslo istorijos vaidmenį fizikos ir matematikos mokymo srityje. Taip pat pateikiu, kai kuriuos samprotavimus apie matematikos, fizikos ir logikos istorijos ir mokymo proceso sąryšį.

*Raktiniai žodžiai:* modeliavimas, matematika, fizika, mokslo istorinė epistemologija.