Volumetric modelling of economic index structures

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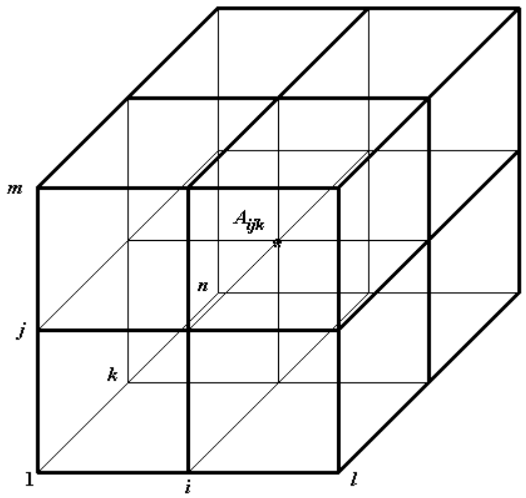
Economic index structures perform very important role in economy management and especially in process of preparing instant decisions. Therefore, timely and reasoned decision making and consequences of their implementation often depends on rationality of such structures. Volumetric model of economic index files, its development and adaptation are described in this article. This mathematical model allows to choose rational structure of particular index file, to evaluate it and to use it in practice gaining particular benefits. Specific example of model appliance is presented.

Preface. One of the most important principles of organizing economic index system is minimization of primary data and maximization of outcome data: as much as possible necessary secondary (derivative) indexes must be obtained from the least quantity of primary indexes, which must be as less duplicated as possible. While implementing this principle, it is very important to choose rational structure of economic index files. Rational structure also allows to process data faster, to manage economy objects and processes more operatively by making timely decisions. Tools of volumetric modelling can be used for rationalization of structures, because analysis of economic indexes shows major usage of indexes that have three requisite-attributes and correspond to volumetric 3-coordinate system in economy and business area. The objective of this article is to show mechanism of volumetric modelling and to show possibilities of model adaptation and usage in practice by choosing and evaluating the most rational economic index structure.

1. Development of model. The framework of volumetric model is shown in pic. As we can see, all file of particular economic index can be imagined as hexahedron, filled in with such values of this index:

$$\{R1_i, R2_j, R3_k, A_{ijk}\};$$

where indexes $i = 1 \div l$, $j = 1 \div m$ and $k = 1 \div n$ indicate values of particular index attributes



*Pic. **Volumetric model of economic indexes***

$R1$, $R2$, $R3$ and basis of particular index A (some part of elements in the space of hexahedron can be blank).

If number of symbols of given indexes is known, then all of its file D size Q can be expressed as sum of its separate parts $D_t\ (t = 1 \div v)$:

$$Q = \sum_{t=1}^{v} \left(al_t + bm_t + cn_t + l_t m_t n_t \right), \qquad (1)$$

if it is assumed, that $Q(A_{ijk}) = 1$, $Q(R1_i) = a$, $Q(R2_j) = b$ and $Q(R3_k) = c$.

According to formula (1) calculated size Q is random quantity as all A_{ijk} are. A_{ijk} values are not equal to zero with probability p_{ijk} and are equal to zero with probability $q_{ijk} = 1 - p_{ijk}$; it means that p_{ijk} is probability of element $\{R1_i, R2_j, R3_k, A_{ijk}\}$ presence in file D and q_{ijk} is probability of its absence. Values of probability p_{ijk} can be considered as given, because we can determine them using analytical method or, for example, with the help of statistical analysis. Also we consider that quantities $p = \sum_{ijk} p_{ijk}$ and $M = lmn$ do not differ much by their sequence, and it means that there is quite strong probability that there will be a lot of non-zero A_{ijk} in file D.

Each part of file D_t can be defined indicating set of hexahedron rectangles I_t, J_t and K_t, where elements A_{ijk} are in intersections of these rectangles. If probability of presence of rectangle with index $i = \in I_t$ is marked $p_{i(t)}$, with index $j = \in J_t$ is marked $p_{j(t)}$, with index $k = \in K_t$ is marked $p_{k(t)}$, and probability of element A_{ijk} presence in file part D_t is marked $p_{ijk(t)}$, then mathematical hopes l'_t, m'_t, n'_t and u'_t of number of rectangles and elements in file part D_t can be expressed in such way:

$$l'_t = \sum_{i \in I_t} p_{i(t)}; \; m'_t = \sum_{j \in J_t} p_{j(t)};$$
$$n'_t = \sum_{k \in K_t} p_{k(t)}; \; u'_t = \sum_{i \in I_t, j \in J_t, k \in K_t} p_{ijk(t)}. \qquad (2)$$

Then we can calculate mathematical hope Q' and dispersion Δ_Q of index file part size $Q(D_t)$ according to these formulas (Венецкий, 1974):

$$\boldsymbol{Q' = \sum_{t=1}^{v} \Bigg(a \sum_{i \in I_t} p_{i(t)} + b \sum_{j \in J_t} p_{j(t)} +}$$
$$\boldsymbol{+ c \sum_{k \in K_t} p_{k(t)} + \sum_{i,j,k} p_{ijk(t)} \Bigg);} \qquad (3)$$

$$\Delta_Q = \sum_{t=1}^{v}\left(a^2 \sum_{i\in I_t} p_{i(t)} \cdot q_{i(t)} + b^2 \sum_{j\in J_t} p_{j(t)} \cdot q_{j(t)} + \right.$$

$$\left. + c^2 \sum_{k\in K_t} p_{k(t)} \cdot q_{k(t)} + \sum_{i,j,k} p_{ijk(t)} \cdot q_{ijk(t)} \right). \quad (4)$$

We can apply such conditions to mathematical hope and dispersion of whole index file size (independently from the way of organizing structure of file):

$$P \le Q' \le (a+b+c+1)lmn; \quad (5)$$

$$0 \le \Delta_Q \le 0{,}25(a^2+b^2+c^2+1)lmn. \quad (6)$$

Now there is a question, how exactly mathematical hope of economic index file size matches its true size. We can express relational size deviation ε from its mathematical hope in such way (Misevičius, 2001):

$$\varepsilon = \frac{|Q-Q'|}{Q'}. \quad (7)$$

If average square deviation is $\sigma_Q = \sqrt{\Delta_Q}$, then ratio between Q and Q' can be expressed like:

$$\frac{|Q-Q'|}{Q'} < \frac{3\sigma_Q}{Q'}. \quad (8)$$

Then we can determine deviation ε without major relational bias according to this formula:

$$\varepsilon = \frac{3\sigma_Q}{Q'}. \quad (9)$$

Therefore, formula (3), when Q' is changed to Q, can be named as model of economic index file, and formula (9) – relational bias of this model.

2. Adaptation of model. Economic indexes can be decomposed into separate parts of hexahedron according to one or two semantic attributes in these ten ways:

1) „zero“ decomposition, when all three attributes are written next to basis of each index (it means that file is not decomposed according to attributes);
2) the file is decomposed into parts according to the first attribute $R1$, value of which is general to all indexes of one part of file, and is written in the heading of this part only once;
3) the file is decomposed similarly according to the second attribute $R2$;
4) the same decomposition according to the third attribute $R3$;
5) indexes are grouped by two attributes at the same time: $R1$ (senior attribute) ir $R2$ (junior attribute); general value of elder attribute is written in the heading of file part only once, and junior attribute is written in subheading only once too;
6) the file is decomposed similarly according to attributes $R1$ and $R3$;
7) the same decomposition according to attributes $R2$ and $R3$;
8) the same decomposition according to attributes $R2$ and $R1$;
9) the same decomposition according to attributes $R3$ and $R1$;
10) the same decomposition according to attributes $R3$ and $R2$.

Now we can educe such calculations for all possible ways of file decompositions from general formulas (2) and (3):

$$Q_1 = (a+b+c+1)\cdot P;$$
$$Q_2 = al' + (b+c+1)\cdot P;$$
$$Q_3 = bm' + (a+c+1)\cdot P;$$
$$Q_4 = cn' + (a+b+1)\cdot P;$$
$$Q_5 = l'(a+bm') + (c+1)\cdot P;$$
$$Q_6 = l'(a+cn') + (b+1)\cdot P;$$
$$Q_7 = m'(b+cn') + (a+1)\cdot P;$$
$$Q_8 = m'(b+al') + (c+1)\cdot P;$$
$$Q_9 = n'(c+al') + (b+1)\cdot P;$$
$$Q_{10} = n'(c+bm') + (a+1)\cdot P, \quad (10)$$

where $l' = l - \sum_{i=1}^{l}\prod_{j=1}^{m} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk}$;

$$m' = m - \sum_{j=1}^{m}\prod_{i=1}^{l} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk} ;$$

$$n' = n - \sum_{k=1}^{n}\prod_{i=1}^{l} q_{ijk} \cdot \prod_{j=1}^{m} q_{ijk} ; \; P = \sum_{i,j,k} p_{ijk} .$$

After calculations according to these formulas, it is possible to determine Q_s with the least value of file size, i.e. the most rational way of organizing index file.

Similarly we educe calculations of dispersion (Kubilius, 1996) from general formula (4):

$$\begin{aligned}
\Delta_{Q_1} &= \left(a^2 + b^2 + c^2 + 1\right) \cdot W ;\\
\Delta_{Q_2} &= a^2 l'' + \left(b^2 + c^2 + 1\right) \cdot W ;\\
\Delta_{Q_3} &= b^2 m'' + \left(a^2 + c^2 + 1\right) \cdot W ;\\
\Delta_{Q_4} &= c^2 n'' + \left(a^2 + b^2 + 1\right) \cdot W ;\\
\Delta_{Q_5} &= l''\left(a^2 + b^2 m''\right) + \left(c^2 + 1\right) \cdot W ;\\
\Delta_{Q_6} &= l''\left(a^2 + c^2 l''\right) + \left(b^2 + 1\right) \cdot W ;\\
\Delta_{Q_7} &= m''\left(b^2 + c^2 l''\right) + \left(a^2 + 1\right) \cdot W ;\\
\Delta_{Q_8} &= m''\left(b^2 + a^2 l''\right) + \left(c^2 + 1\right) \cdot W ;\\
\Delta_{Q_9} &= n''\left(c^2 + a^2 l''\right) + \left(b^2 + 1\right) \cdot W ;\\
\Delta_{Q_{10}} &= n''\left(c^2 + b^2 m''\right) + \left(a^2 + 1\right) \cdot W , \qquad (11)
\end{aligned}$$

where

$$l'' = \sum_{i=1}^{l}\left(1 - \prod_{j=1}^{m} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk}\right) \cdot \prod_{j=1}^{m} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk} ;$$

$$m'' = \sum_{j=1}^{m}\left(1 - \prod_{i=1}^{l} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk}\right) \cdot \prod_{i=1}^{l} q_{ijk} \cdot \prod_{k=1}^{n} q_{ijk} ;$$

$$n'' = \sum_{k=1}^{n}\left(1 - \prod_{i=1}^{l} q_{ijk} \cdot \prod_{j=1}^{m} q_{ijk}\right) \cdot \prod_{i=1}^{l} q_{ijk} \cdot \prod_{j=1}^{m} q_{ijk} ;$$

$$W = \sum_{i,j,k} q_{ijk} \cdot p_{ijk} .$$

After selecting Δ_{Q_s}, we can determine relational bias of particular model of economic index file according to formula (9):

$$\varepsilon = \frac{3\sqrt{\Delta_{Q_s}}}{Q_s} . \qquad (12)$$

Now we calculate relational economy, which is obtained choosing the most rational structure:

$$\mathrm{E} = \left(\frac{\overline{Q} - Q_s}{\overline{Q}} - \varepsilon\right) \cdot 100 ,$$

where $\overline{Q} = \frac{1}{10}\sum_{r=1}^{10} Q_r$. (13)

$\overline{Q}$ can be considered as mathematical hope of index file size, when the way of organizing it is chosen randomly.

3. Model in practice and conclusions. It is always possible to determine level of filling hexahedron with indexes by analytical method, and all p_{ijk} values, as a rule, are the same. Therefore, while changing p_{ijk} to p, and q_{ijk} to q, we can express formulas (10) like that:

$$l' = l\left(1 - q^m \cdot q^n\right) ; \; m' = m\left(1 - q^l \cdot q^n\right) ;$$

$$n' = n\left(1 - q^l \cdot q^m\right) ; \; P = lmnp . \qquad (14)$$

We change formulas (11) similarly:

$$l'' = l\left(1 - q^m \cdot q^n\right) \cdot q^m \cdot q^n ;$$

$$m'' = l\left(1 - q^l \cdot q^n\right) \cdot q^l \cdot q^n ;$$

$$n'' = n\left(1 - q^l \cdot q^m\right) \cdot q^l \cdot q^m ; \; W = lmnpq . \qquad (15)$$

We will illustrate specific example of selection of one rational economic index file structure. We have to find the best variant of investment distribution file according to projects, managers and periods. Its index structure is showed in table 1.

Table 1. ***Structure of index***

Length of requisite (number of symbols)	*Requisite*	
	Label	***Title***
3	$R1_i$	Project
4	$R2_j$	Manager
3	$R3_k$	Period
5	A_{ijk}	Sum of assigned finances

Table 2. ***Primary data of calculations***

a	***b***	***c***	***l***	***m***	***n***	***p***	***q***
0,6	0,8	0,6	30	50	36	0,16	0,84

Suppose that $Q(A_{ijk}) = 1$. Then $a = Q(R1_i) = 0{,}6$; $b = Q(R2_j) = 0{,}8; c = Q(R3_k) = 0{,}6$. Suppose that attribute $R1_i$ has 30 values (there are 30 projects running; $l = 30$), there are 50 managers ($m = 50$), and there are 36 periods, for example, number of 3 year period months ($n = 36$). Although practically there are only 12-20 managers in one project and finances are assigned to 12-24 periods. We will use average values in calculations, i.e. respectively 16 and 18. We determine from attributes that probability of filling hexahedron by sums of finances is equal 0,16. All data required for calculations is showed in table 2.

Using formulas (14) according to formulas (10) we calculate values of all methods of file organization Q:

$Q_1 = 25.920$; $Q_2 = 20.750$; $Q_3 = 19.048$; $Q_4 = 20.758$; $\underline{Q_5 = 14.562}$; $Q_6 = 16.218$; $Q_7 = 14.944$; $Q_8 = 14.764$; $Q_9 = 16.222$; $Q_{10} = 15.286$.

Consequently the least value is Q_5, and it means that the most rational method of file organization is the fifth one, when values of index basis are grouped by two attributes: project (elder) and manager (junior) and the period is written by every value of assigned finances sum.

When we define data required for further calculations according to formulas (15), we calculate that dispersion $\Delta_{Q_5} = 9.872$ according to formula (11), and we calculate what bias of our model $\varepsilon = 0{,}02$ according to formula (12). Eventually according to formula (13) we calculate obtainable relational economy: $E = 16{,}4\ \%$. As we can see, volumetric modelling of index file produces fair benefits.

It is necessary to point out that all calculations that are related with development and evaluation of economic index file model can be done automatically by using specific computer programme.

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TŪRINIS EKONOMINIŲ RODIKLIŲ STRUKTŪRŲ MODELIAVIMAS

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Santrauka

Ekonominių rodiklių struktūros atlieka labai svarbų vaidmenį ūkio valdyme ir ypač – skubiai rengiant sprendimus. Todėl nuo tų struktūrų racionalumo labai dažnai priklauso pagrįstų sprendimų priėmimas laiku ir jų įgyvendinimo rezultatai. Straipsnyje aprašomas tūrinis ekonominių rodiklių rinkmenų modelis, jo kūrimas ir adaptavimas. Šis matematinis modelis leidžia išrinkti racionalią konkretaus rodiklio rinkmenos struktūrą, ją įvertinti ir taikyti gaunant tam tikrą naudą. Pateikiamos rodiklių rinkmenos dalies apimties matematinės vilties ir dispersijos skaičiavimo formulės, taip pat parodomi skaičiavimai visiems galimiems rinkmenos skaidymo būdams. Pateikiamas konkretus modelio naudojimo pavyzdys.

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